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ORIGINAL RESEARCH ARTICLE

Lattice Connectivity and Entanglement in Quantum Spin-glasses

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ABSTRACT

I have studied the role of lattice connectivity and coupling weights distribution on the entanglement of quantum spin-glasses. Its found in this work that the connectivity of the lattice weakly influence the degree of entanglement in the spin-glass compared to the distribution of the coupling constants between the spins. This suggest important implications for machine learning models such as Boltzmann machines and the study of complex quan-tum systems.

INTRODUCTION

The study of spin glasses is a fascinating and highly interdisciplinary field that lies at the intersection of statistical mechanics, condensed matter physics, and computer science (Bapst et al., 2013; Georges et al., 2000; Hen et al., 2015; Kopeć & Usadel, 1997). Spin glasses are disordered magnetic materials that exhibit complex behaviour due to the presence of many competing and frustrated interactions between the magnetic moments or spins of the individual atoms or molecules (Kopeć & Usadel, 1997).

Lattice connectivity and entanglement are both important concepts in the study of quantum spin-glasses. In a spin-glass, a large number of interacting spins are arranged on a lattice, and the interactions between the spins are random and frustrated, leading to the emergence of a complex and disordered ground state (Bapst et al., 2013; Grest et al., 1986; C. R. Laumann et al., 2010).

Lattice connectivity refers to the way in which the spins are connected on the lattice. For example, a spin-glass may have a regular lattice structure, such as a square or triangular lattice, or it may have a more complex structure, such as a random graph or a fractal lattice. The connectivity of the lattice can have a significant impact on the properties of the spin-glass, such as its critical temperature and the nature of its ground state. This work considers three lattice connectivities: fully connected lattice, bi-partite graph, and a ring (see the sketch in Figure 1).

Entanglement, on the other hand, refers to the correlations between different spins in the spin-glass (Côté & Kourtis, 2022; Koh, 2014; Koh & Kwek, 2014; Yatsuzuka et al., 2022). In a quantum spin-glass, the spins are described by quantum mechanical wavefunctions, and the interactions between the spins can lead to entanglement between them. Entanglement is a fundamental feature of quantum mechanics, and it can have a significant impact on the properties of the spin-glass, such as its quantum criticality and the nature of its ground state.

In this article, we study of the relationship between lattice connectivity and entanglement in quantum spin-glasses. This research has revealed that the connectivity of the lattice can have a significant impact on the degree of entanglement in the spin-glass, measured by the von Neumann entropy definition. This research has important implications for the design and optimization of quantum inspired machine learning models, as well as for the development of new theoretical tools and techniques for the study of complex quantum systems.

METHODOLOGY

In this section, we will outline the model definition for quantum spin-glass, describe the simulation procedure and data analysis.

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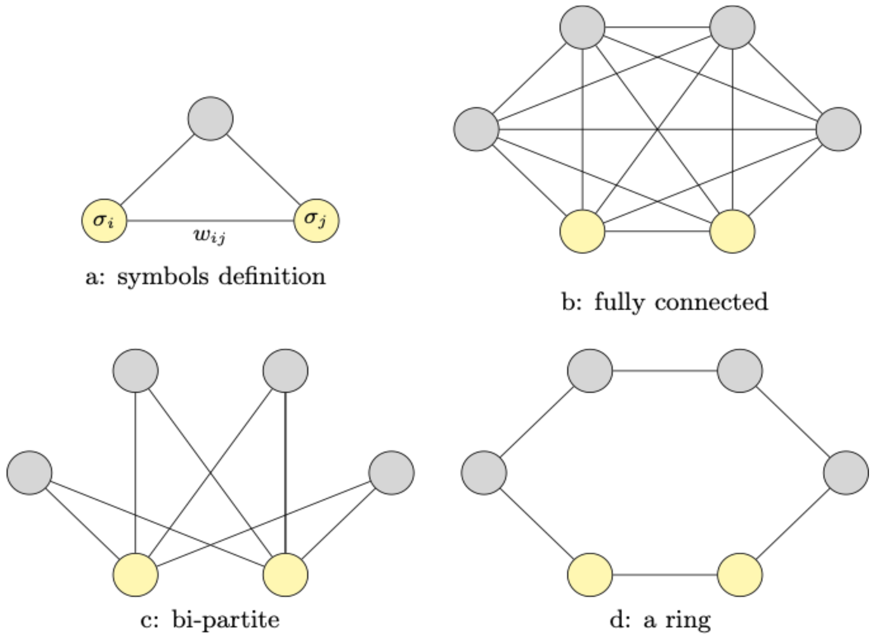

Figure 1: Sketch showing some graph connectivities and definition of symbols.

The model

The energy of the spin glass system with spin-spin interactions w_{ij} between σ_i and σ_j is:

$$H(\sigma) = -\sum_{i,j=1}^{n} w_{ij}\,\sigma_i^z \sigma_j^z - \sum_{j=1}^{n} h_j\,\sigma_j^x, \qquad (1)$$

Where $\sigma_j^x = 1 \otimes \cdots \otimes \sigma^x \otimes \cdots 1$ and σ^x is at the j'th location, $w_{i,j}$'s are the weights (or coupling constants), h_j's are external field (or bias).

The quantum channel is therefore given by:

$$U(\sigma) = e^{-iH(\sigma)t/\hbar}, \qquad (2)$$

where we consider $\hbar$ is the Planck's constant and t is time (see Figure 2). Therefore, the density matrix describing the evolution of the system, which will be used to determine the von Neumann entropy in the next section, is:

$$\rho = U^{\dagger}|0 \cdots 0\rangle\langle 0 \cdots 0|U, \qquad (3)$$

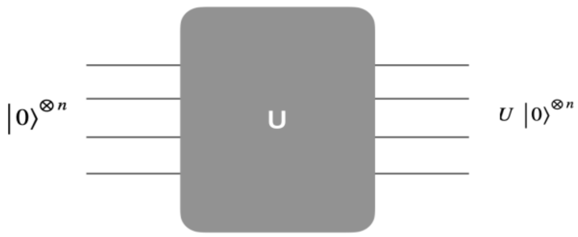

Figure 2: Quantum circuit showing the unitary gate U as grey box and the wires represent the individual quibits.

Entanglement entropy measure

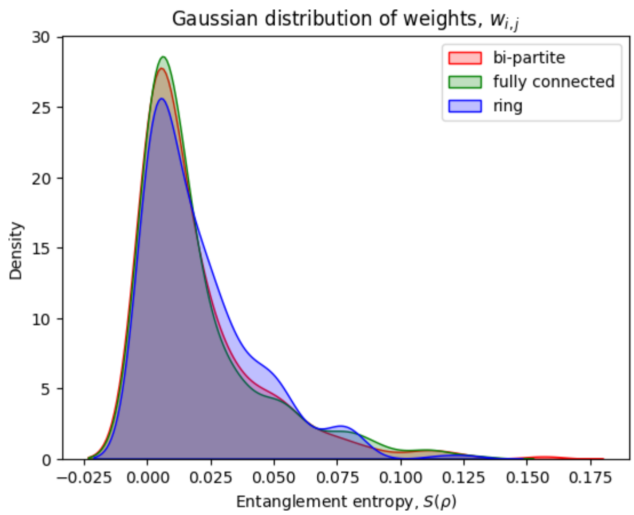

Figure 3: Entanglement entropy distribution for a normally distributed lattice weights.

We can measure the degree of entanglement with the von Neumann entropy, $S(\rho)$, defined

$$S(\rho) = -Tr(\rho log \rho), \qquad (4)$$

where 'Tr' is the trace operator.

Numerical implementation

These quantum channel in eqn. (2) is evolved for unit time (unit of $\hbar$). The results of the theoretical and numerical calculations are plotted in Figures. (3) and (4). We will next discuss these results highlighting their implications.

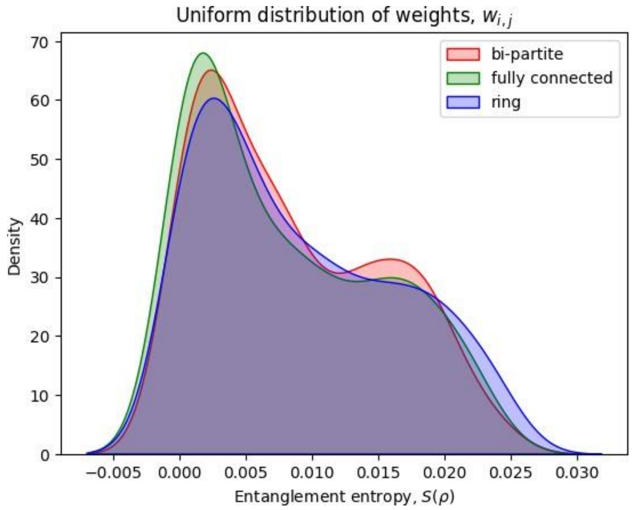

Figure 4: Entanglement entropy distribution for a uniformly distributed lattice weights.

RESULTS AND DISCUSSION

Analysis of the simulation data shown in Figure 3 and Figure 4 suggests the following:

1.) the degree of entanglement in spin-glasses depends only weakly on the lattice connectivity

2.) the system entanglement shows strong sensitivity to the distribution of the coupling constants.

The two results above indicate that the behaviour of entanglement in spin-glasses is affect-ed by different factors to different degrees. The first result, which shows that the degree of entanglement in spin-glasses depends only weakly on the lattice connectivity, implies that the way the spins are connected in the lattice has a limited effect on the entanglement properties of the spin-glass (see Figures 3 and 4). Despite the difference in the lattice connectivity, from fully-connected to a ring, the information content distribution in the system dynamics remain almost identical. This result suggests that other factors such as the distribution of coupling constants, as indicated in the second result, may play a more significant role in determining the entanglement behavior of the system.

The second result, which shows that the system entanglement is strongly sensitive to the distribution of the coupling constants, implies that the strength of the interactions between the spins is a crucial factor in determining the entanglement properties of the system. The coupling constants determine the strength and type of interaction between the spins, and a specific distribution of coupling constants can lead to the emergence of specific entanglement properties in the spin-glass.

Taken together, these results highlight the complex interplay between different factors in determining the behavior of entanglement in spin-glasses. While the connectivity of the lattice may have some impact on the system's entanglement, the distribution of coupling constants seems to be a more significant factor in determining the entanglement behaviour of the system. These findings could have important implications for the design and optimization of quantum-inspired machine learning models and the development of new theoretical tools and techniques for the study of complex quantum systems.

Previous studies of quantum spin-glasses tends to focus extensively on Bethe lattices (Kopeć & Usadel, 1997; C. Laumann et al., 2008; Mossi et al., 2017), perhaps because of the successful analytical approximation technique of Bethe ansatz in solving these systems. While this is important to investigate, understanding general relationship between lattice connectivity and the system collective behaviour will require ensemble approach that we have adopted in this work.

CONCLUSION

We have studied the role of lattice connectivity in the entanglement of quantum spin-glasses. We have found that its the distribution of the coupling constants rather than the lattice connectivity that determines the degree of entanglement in a spin-glass system. The results suggests that for the vast majority of the set of coupling constants mediating the interactions between the spins, the spins are weakly entangled. The system shows strong sensitivity to distribution of the connectivity weights (or coupling constants) compared to underlying graph connectivity between the spins (see Figure 3 and 4). While we study small system sizes due to computational resources constraint, we are confident our results holds true in the large system size limit. Future work would focus on studying large sys-tem sizes and exploring large parameter space and phase transitions.

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