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ORIGINAL RESEARCH ARTICLE

Graph Coloring with inversion in the Γ_1 non-Deranged Permutations

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ABSTRACT

In this paper, we investigate graph coloring with inversion in Γ_1 non-deranged permutations, the permutation which fixes the first element in the permutations.This was first accomplished by performing some calculations on this strategy using prime numbers. $p \geq 5$. As a result, We observed that the chromatic number of any $\chi(G(\omega_{P-I}))$ in $G_p^{\Gamma_1}$ is equal to $p-1$ and any $\chi(G(\omega_I))$ in $G_p^{\Gamma_1}$ is equal to one. Similarly, the chromatic index of any $\chi'(G(\omega_{P-I}))$ in $G_p^{\Gamma_1}$ is equal to $p-2$ and any $\chi'(G(\omega_I))$ in $G_p^{\Gamma_1}$ is equal to zero. Results for this investigation established that chromatic number and chromatic index are related.

INTRODUCTION

An inversion of the permutation f is a pair (i, j) such that $i < j$ and $f_i > f_j$ which is denoted as $Inv(\omega_i)$ and the number of inversion f denoted by $inv(\omega_i) = |Inv(\omega_i)|$ .Also a basic graph is colored when each vertex is given a different color, ensuring that no two neighboring vertices have the same color. Aunu permutation pattern first arose, out of attempts to provide some combinatorial interpretations of some succession scheme and today series of results ranging from its avoiding class to its properties which have also been studied. Garba and Ibrahim (2010) developed a strategy for the prime numbers $p \geq 5$ and $\Omega \subseteq N$ employing the catalan numbers as well. This scheme creates a cycle of permutation patterns that is utilized to decide the arrangements. Researchers have over time looked also at permutation group with certain properties; one that comes to mind is the permutation patterns that have any of the element fixed or the one that has no fixed element, here the idea of deranged and non-deranged permutation surface. It is in line with this understanding that Ibrahim *et al.,*(2016) modified the scheme of Garba and Ibrahim (2010) to two linenotation and the scheme generated a set of permutations with a fix at 1 (which generated the natural identity). This obtained set of permutations form permutation groupcalledthe Γ_1 non -deranged permutation group and is denoted as $G_p^{\Gamma_1}$ .Ibrahim *et al.,* (2017) outline the theoretical characteristics of the Ascent set in regard to the Γ_1 non- deranged permutation and demonstrate that the union of the Ascent set equals the identity. They also note that the difference between $Asc(\omega_i)$ and $Asc(\omega_{p-1})$ is one.Aremu *et al.,* (2018) utilized the direct and skew sum operation on the components of the Γ_1 non- deranged permutation group and showed the relationships and schemes on the structures and fixed point of the permutations generated from these operations

Additionally, if π isthe direct sum of these Γ_1 non-deranged permutations, then the collection of permutations in the form of π is an abelian group, designated as $G_p^{\Gamma_{m\oplus}}$. According to Aremu *et al.*, (2019), the $Res(\omega_i)$ and $Res(\omega_{p-1})$ of Γ_1 non-deranged permutations are equally distributed between the right and left embracing numbers, respectively. Additionally, it notices that the height of the weighted motzkin path of $\omega_i$is the same height as the height of $\omega_{p-des(\omega_i)}$ motzkin

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path. Ibrahim and Garba (2019) studied ascent and descent blocks of Γ_1 - non deranged permutations. moreover, it defined the mapping $\Psi_{AI}: G_p^{\Gamma_m} \to \Omega_p$ which converts the permutation from the weighted Motzkin path in the Ω_p with respect to the ascent and descent blocks from the Γ_1 non- deranged permutations group $G_p^{\Gamma_m}$. Inversion number and major index are not equally distributed in Γ_1 non- deranged permutations, as demonstrated by Garba and Ibrahim (2019), who also established that the difference between the sums of the major index and the inversion numbers is equal to the sum of the descent numbers in these permutations. Ibrahim and Muhammad (2019) produced recursion formulas for the maximum and lowest block counts in Γ_1- non deranged permutations. They also noted that $arc(\omega_i)$ and $asc(\omega_i)$ are equally distributed in Γ_1 non- deranged permutations. Using the membership function that was built for the fuzzy set on G_p', Ibrahim *et al.,* (2021) investigated some algebraic theoretic properties of the fuzzy set on G_p' and established the results for the algebraic operators of the fuzzy set on G_p', which are the algebraic sum, algebraic product, bounded sum, and bounded difference. They also built a relationship between the operators and the fuzzy set on $\mathcal{G}_p'$. More recentlyIbrahim *et al.,*(2022) proved that the right embracing sum $\mathrm{Re}s(\omega_i)$ is equal to the right embracing sum $\mathrm{Re}s(\omega_{p-i})$ where $1 \le i \le p-1$ and $\mathrm{Re}s(\omega_i) = \frac{(p-3)(p-1)}{8}$ where $i = \frac{p-1}{2}, \frac{p+1}{2}$. It also observed that the left embracing sum $Les(\omega_i)$ is equal to the right embracing sum $\mathrm{Re}s(\omega_i)$ and $Les(\omega_{\frac{p+1}{2}}) = \frac{p^2-1}{8}$ where $p \ge 5$.

In order to prevent neighboring vertices from having the same color, a graph is colored by giving a different color to each of its vertices. To produce a suitable color that can be employed in relation to inversion statistics in chromatic index and chromatic number is the goal of this study.

MATERIALS AND METHODS

In this section before we outline the main results in this research paper, we attempt to define some basic concept that will help in further understanding of this work.

Γ_1 non- deranged permutations

Let Γ be a non-empty set of prime cardinality $p \ge 5$ such that $\Gamma \subset N$. A bijection ω on Γ of the following kind.

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mop} & (1+2i)_{mop} & . & . & . & (1+(p-1)i)_{mop} \end{pmatrix}$$

Also known as Γ_1 non- deranged permutations. The set of all regular, Γ_1-non deranged permutations is designated as G_p.

Γ_1 non- deranged permutations group

The group $G_p^{\Gamma_1}$ is formed from the pair G_p and the composition of natural permutations. This unique permutation group fixes the initial component of Γ.

Inversion

An inversion in permutation $\omega_i = a_1 a_2 \ldots a_n$ is a pair (i, j) such that $i < j$ and $a_i > a_j$ which is denoted as $Inv(\omega_i)$ and the number of inversion ω_i denoted by $inv(\omega_i) = |Inv(\omega_i)|$

Graph

A graph $G = (V, E)$ is a pair, where V denotes the set of vertices and E denotes the set of edges (not necessary non-empty)

Connected Graph

If a path connects any two vertices in a graph, then the graph is said to be connected. If not, the graph is not connected.

Complete Graph

A complete graph is one in which every set of unique vertices is adjacent.

Null Graph

If the edge set of a graph G is empty, it is referred to as a null graph and is indicated by the symbol N_n.

Matching

A group of edges in a graph that do not share any vertices is known as a matching or independent edgeset.

Matching Number

The number of edges in the maximal matching is known as the matching number.

Regular Graph

If every vertex in a graph G has an equal number of degrees, then the graph is considered regular.

Graph Coloring

A graph is colored by giving a different color to each of its vertices so that no two neighboring vertices have the same color

RESULTS AND DISCUSSION

In this section, we discuss the investigations' details and results obtained.

Proposition 1

Let G_{ω_i} be Γ_1-non deranged permutations of subgroup of ω_i, then every Γ_1 non- deranged permutations of G_{ω_i} where $i \neq 1$ has to connected component, that is $c(G) = 2$.

Proof:

Since all the permutation graph of Γ_1 non- deranged permutations has an isolated vertex and a path exists between each vertex remaining in the graph, then this shows that every Γ_1-non deranged permutation graph has 2 connected component.

Proposition 2

Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ_1 non- deranged permutation graph of ω_i and ω_{p-i} respectively, then the permutation graph of ω_i and ω_{p-i} has no common edges.

Proof:

ω_i and ω_{p-i} of Γ_1 non- deranged permutations are inverse of each other and it is immediate to see it in their permutation graph structure that they have no common edges.

Remark 3

Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ_1 non- deranged permutation graph of ω_i and $G_{\omega_{p-i}}$ respectively, then

I. $G_{\omega_i} - \{v_1\}$, where $i \neq 1$ is connected graph

II. $G_{\omega_{p-1}} - \{v_1\}$, is a complete connected regular graph

III. G_{ω_i} is a null graph, if $i = 1$.

Proposition 4

Let G_{ω_i} be Γ_1 non- deranged permutation graph of ω_i and $G_{\omega_{p-1}} - \{v_1\}$ denoted permutation induced sub graph of ω_1 then

$$G_{\omega_{p-1}} - \{v_1\} \cong K_{p-1}$$

Proof:

A complete graph K_n has $\frac{n(n-1)}{2}$ edges and since G_{ω_i} has p vertices, then by Remark 3.3 $G_{\omega_{p-1}} - \{v_1\}$ is a complete graph with $p-1$ vertices. The number of edges of $G_{\omega_{p-1}} - \{v_1\}$ is $\frac{(p-1)(p-2)}{2}$, then the permutation graph $G_{\omega_{p-1}} - \{v_1\}$ is isomorphic to K_{p-1}.

Proposition 5

Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ_1 non- deranged permutation graph of ω_i and ω_{p-i} respectively, then

$$\left(G_{\omega_i} \cup G_{\omega_{p-i}}\right) - \{v_1\} \cong K_{p-1}$$

Proof:

Suppose $G_{\omega_i} - \{v_1\}$ and $G_{\omega_{p-i}} - \{v_1\}$ denote the permutation induced sub graph of G_{ω_i} and $G_{\omega_{p-i}}$ respectively, such that

$$G_{\omega_i} - \{v_1\} \cup G_{\omega_{p-i}} - \{v_1\} = \left(G_{\omega_i} \cup G_{\omega_{p-i}}\right) - \{v_1\}$$

$$= G_{\omega_{p-i}} - \{v_1\}$$

$$\cong K_{p-1}$$

Proposition 6

Let G_{ω_i} and $G_{\omega_{p-i}}$ be Γ_1 non- deranged permutation graph of ω_i and ω_{p-i} respectively, then for every permutation graph of G_{ω_i} is spanning sub graph of the permutation graph $G_{\omega_{p-i}}$.

Proof:

The graph order $\left|V\left(G_{\omega_i}\right)\right|$ and $\left|V\left(G_{\omega_{p-i}}\right)\right|$ are equal and the graph size $\left|E\left(G_{\omega_i}\right)\right|$ is less than the graph size of $\left|E\left(G_{\omega_{p-i}}\right)\right|$ the result follows.

Proposition 7

Let G_{ω_i} be Γ_1 non- deranged permutation graph of ω_i where $1 \le i \le p-1$ and $\alpha^{'}$ denoted the matching number of the permutation graph G_{ω_i} then

$$\alpha^{'}\left(G_{\omega_i}\right) = \frac{(p-1)}{2}$$

Proof:

Suppose G_{ω_i} be Γ_1 non-deranged permutation graph of ω_i and $G_{\omega_i} - \{v_1\}$ denoted the permutation induced the permutation sub graph of G_{ω_i} since $G_{\omega_i} - \{v_1\}$ is connected graph, then the order will be $\left|G_{\omega_i} - \{v_1\}\right| = p-1$ and half of $p-1$ gives the matching number of the permutation graph G_{ω_i} for every i.

Proposition 8

Let $G_{\omega_{p-1}}$ be Γ_1 non- deranged permutation graph of ω_{p-1} and $\chi(G(\omega_{p-1}))$ denoted the chromatic number of the permutation graph $G_{\omega_{p-1}}$ then

$$\chi(G(\omega_{p-1})) = p-1$$

Proof:

By Remark 3(ii) for any $\omega_{p-1} \in G_p^{\Gamma_1}$, the graph $G_{\omega_{p-1}} - \{1\}$ is complete. Therefore the chromatic number of the graph of ω_{p-1} is $p-1$.

Proposition 9

Let G_{ω_1} be Γ_1 non- deranged permutation graph of ω_1 and $\chi(G(\omega_I))$ denoted the chromatic number of the permutation graph ω_1 then

$$\chi(G(\omega_1)) = 1$$

Proof:

By Remark 3(iii) for any $\omega_1 \in G_p^{\Gamma_1}$, the graph G_{ω_1} is null/empty. Then thereis no edge in the graph (each vertex is isolated) and hence the chromatic number of the graph of $G_p^{\Gamma_1}$ is 1.

Proposition 10

Let $G_{\omega_{p-1}}$ be Γ_1 non- deranged permutation graph of ω_{p-1} and $\chi'(G(\omega_{p-1}))$ denoted the chromatic index of the permutation graph $G_{\omega_{p-1}}$ then

$$\chi'(G(\omega_{p-1})) = p-2$$

Proof:

By Remark 3(ii) for any $\omega_{p-1} \in G_p^{\Gamma_1}$, the graph $G_{\omega_{p-1}} - \{1\}$ is complete.Choose any edge ein the graph $G_{\omega_{p-1}} - \{1\}$, Supposethefirstcolorisassignedtotheedge e .Paintalltheedgesadjacentto e with second color.Next paint these edges adjacent to this using firstcolor. Continue this process till every edge in the graph $G_{\omega_{p-1}} - \{1\}$ has painted. Hence the chromatic index of the graph of ω_{p-1} is $p-2$.

Proposition 11

Let G_{ω_1} be Γ_1 non- deranged permutation graph of ω_1 and $\chi'(G(\omega_1))$ denoted the chromatic index of the permutation graph ω_1 then

$$\chi'(G(\omega_1)) = 0$$

Proof:

For any $\omega_1 \in G_p^{\Gamma_1}$ by Remark 3.3(iii), the graph ω_1 is empty or null. As a result, the graph has no edges, and as a result, its chromatic index is equal to zero.

CONCLUSION

In this paper, we compute the graph coloring using inversion on Γ_1 non -deranged permutations.

We found that every $\chi(G(\omega_{p-1}))$ in $G_p^{\Gamma_1}$ has a chromatic number of $p-1$ and any $\chi(G(\omega_1))$ in $G_p^{\Gamma_1}$ has a chromatic number of 1. Similar any $\chi'(G(\omega_{p-1}))$

in $G_p^{\Gamma_1}$ has a chromatic index of $p-2$ and any $\chi'(G(\omega_1))$ in $G_p^{\Gamma_1}$ has a chromatic index of zero.In order to discover new graphic and combinatorial results, more research on Γ_1 non- deranged permutations should be done in relation to other permutation statistics like fixed point, Record, Anti record, Cycle valley, and others.

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