UMYU Scientifica, Vol. 1 NO. 2, December 2022, Pp 98 – 102
https://doi.org/10.56919/usci.1222.012
ISSN: 2955 – 1145 (print); 2955 – 1153 (online)
A periodical of the Faculty of Natural and Applied Sciences, UMYU, Katsina

ORIGINAL RESEARCH ARTICLE

Comparative Study of Lee Carter and Arch Model in Modelling Female Mortality in Nigeria

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ABSTRACT

Using Nigeria mortality data from 2009 to 2020, this study compares and contrasts how well the Lee-Carter and ARCH models performed. Singular value decomposition (SVD) method, Langrage multiplier test, and autoregressive conditional heteroskedasticity (ARCH) effects were examined. Five (5) different ARIMA and ARCH models were fitted together with their criteria, i.e., AIC and BIC in order to determine the best model for Nigeria mortality data. ARIMA (0,1,0) had the lowest AIC and BIC values, and was determined to be the best ARIMA model. The mortality index is then modelled using ARIMA (0,1,0) and plugged back into the Lee-Carter model to predict the future mortality rate. Also ARCH (1) turned out to be the best ARCH model among other candidate models. The performance of Lee-Carter model and ARCH model was compared using error measures. Results obtained revealed that the ARCH model had the minimum RMSE and MAPE when compared with the Lee-carter model, therefore it was concluded that the ARCH model performs better than the Lee-Carter model on Nigeria mortality data.

ARTICLE HISTORY

Received September 13, 2022
Accepted December 20, 2022
Published December 30, 2022

KEYWORDS

ARCH, Lee-Carter model, Mortality, Singular value decomposition.

INTRODUCTION

Every nation globally whether developed or developing must strive for its citizens' prosperity. Every nation, therefore, needs to understand its population's size, composition, and changes. Demography is the study of a population's size, composition, and changes over time. At present, the mortality rate has decreased as a result of technological developments in medicine and public health. Mortality rates are frequently used in other fields as well. For instance, actuaries and public health professionals in government utilize life tables to determine the population's average lifetime and longevity. To compute insurance premiums and reserve needs, actuaries must also be able to construct a precise mortality table.

The government can evaluate population policy measures and demographic projections that will ultimately be used for national development goals using mortality rates. Corporations have had to deal with two main forms of risk when offering life insurance contracts i.e. demographic and financial risk. Ambiguity over eventual death presents a barrier for financial organizations that offer annuities and other life-dependent products. Various efforts have been made to look for an acceptable model that can reflect mortality in demography.

Lee and Carter (LC) model developed in 1992 is one of the most frequently used models for predicting age specific death rates. The model has garnered a lot of attention and become a standard for estimates of life tables and death rates, but it has also come under criticism. Although the model's original purpose was to describe the variation in all-cause mortality in the US and other advanced nations, it is now frequently used to project death from all causes and specific causes for many countries over a variety of time periods. In Japan (Wilmoth, 1993), Austria (Carter and Prskawetz, 2001), Australia Booth *et al.,* 2002), and Belgium (Tuljapurkar, 2008), for example, the Lee-Carter model was used. Despite being widely used and performing adequately, the Lee-Carter model has several drawbacks (Lee, 2000). Researchers like Gutterman and Vanderhoof (1999) have criticized these shortcomings. Given that the authors might not have anticipated some of these limitations, it seems like a good moment to compare the Lee-Carter strategy with alternative approaches.

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How to cite: Shelleng, A.U., Sule, Y.J., Kajuru, J.Y., and Kabiru, A. (2022). Comparative Study of Lee Carter and Arch Model in Modelling Female Mortality in Nigeria. UMYU Scientifica, 1(1), 241 – xxxx. https://doi.org/10.56919/usci.1222.012

One unique feature of the Lee-Carter approach is that the mortality index k_t is modelled using a time series approach (Brockwell and Davis, 1996). In the past, Lee and Carter, and many other researchers have utilized an *ARIMA (0,1,0)* with drift. Although the ARIMA model is frequently used in the Lee-Carter approach, its suitability for forecasting noisy and nonlinear data (such as mortality data) has been questioned. Therefore, the study seeks to examine and compare the forecasting efficiency of Lee-Carter model and ARCH model on mortality data in Nigeria. This can be achieved through the following objectives:-

i. Model Nigeria mortality data using the Lee-Carter model.
ii. Model Nigeria mortality data using the ARCH model
iii. Compare the performance of the models using error measures

MATERIALS AND METHODS

Data

The data for this research was obtained from the obtained from the WHO Indicator and Measurement Registry (IMR). These data obtained was recorded over the period of 2009 to 2020 covering Nigerian female mortality rates. R package version 4.1.2 was used for the Analysis

Lee-Carter model

In 1992, Lee and Carter put forth a technique for predicting death rates. The base model and the ARIMA model are the two essential components of the Lee-Carter method. There are no subjective assessments included in the projection because it is based on a stable long-term trend. In particular, the LC model does forecasting in two stages. In the first stage, a_x, b_x *and* k_t were estimated using singular value decomposition method (SVD). In the second stage, the Box and Jenkins (1970) approach is used to model and extrapolate the fitted values of k_t using an autoregressive integrated moving average process.

$$ln(m_{xt}) = a_x + b_x k_t + \varepsilon_{xt} \quad (1)$$

Where, the matrix of the actual age specific death rate at age x during year t is given as $m_{(xt)}$

a_x is the general pattern of death by age

b_x is the rate of change in death as a function of age as k_t changes. This explains the shape of age profile deviations when the parameter k_t is altered

k_t is the general mortality trend throughout time. The mortality index is another name for it. The values of the variable k_t is a time series since it is actually considered primarily as a stochastic process (rather than a deterministic quantity).

ε_{xt} is the error term at age x and time t.

Singular Value Decomposition

Bell and Monsell (1991) pioneered the Singular Value Decomposition (SVD) method. The technique exposes many of the essential and distinctive characteristics of the original matrix by dissecting a matrix into its many sub matrices. Orthogonal matrix U, a diagonal matrix D, and the transpose of an orthogonal matrix V are the three matrices that can be combined to form a matrix, according to a linear algebraic theorem that serves as its basis. The matrix is divided into several factors that, are orthogonal and independent.

A real $m \times n$ matrix A can be uniquely decomposed as:

$$A = UDV^T \quad (2)$$

A matrix A is assumed to be of size $m \times n$. U matrix is of size $m \times m$, matrix D is of size $m \times n$, and matrix V is define to be of size $n \times n$.. All of these matrices are well-defined to have a discrete structure. Matrix U and matrix V are both said to be orthogonal. The matrix D is define as a diagonal matrix. All the elements on the diagonal of D are regarded as the singular values of the matrix A. Columns of matrix U are regarded as the left singular vectors. The right-singular vectors are the columns of V.

The Lee-Carter model cannot be fitted using the traditional regression technique since no regressors are given on the right side of the estimate. a_x, β_x and k_t are estimated and denoted by $\hat{a}_x, \hat{\beta}_x$ and $\hat{k}_t$ separately. Firstly, let's define the $\hat{a}_x$ estimate as the mean of $\ln(\hat{m}_{x,t})$ progressively over time t

$$\hat{a}_x \frac{1}{n} = \sum \ln m_{xt} \quad (3)$$

Parameters β_x and k_t are estimated by applying SVD on the matrix Z.

here;

$$z_{xt} = \ln m_{xt} - \hat{a}_x \quad (4)$$

That is;

$$SVD(Z_{xt}) = ULV \quad (5)$$

Here U denotes the age component is denoted by U the singular values denoted by L and the time denoted by V.

Hence after decomposition,

$$\text{SVD}(Z_{x,t}) = \lambda_1 U_{x,1} V_{t,1} + \lambda_2 U_{x,2} V_{t,2} + \cdots + \lambda_i U_{x,i} V_{t,i} = \sum_{i=1}^{r} \lambda_i U_{x,i} V_{t,i} \quad (6)$$

Where r = Rank $|Z|$ and $\lambda_i (i = (1,2,\ldots,r)$ are the ordered singular values while $V_{t,i}$ and $U_{x,i}$ are the right and left singular vectors. These give the estimates of b_x *and* k_t as follows;

$\hat{\beta}_x = U_{x,1}$ And $\hat{k}_t = \lambda_1 V_{t,1}$ $\hat{b}_x = U_{x,1}$ and $\hat{k}_t = V_{t,1}$

Arch model

The Autoregressive Conditional Heteroskedasticity method is a method for describing a change in variance for a time-dependent time series, such as rising or declining volatility. Varying volatility is referred to as "heteroskedasticity." An ARCH model assumes that rather than the variance itself, the conditional variance, which depends on the data available evolves over time. The conditional variance estimates how definite we can be about the future observation. *ARCH (q)* model for the series $\{\varepsilon_t\}$ is well-defined by determining the conditional distribution ε_t given the available information up to time $t-1$. Let's use the term ψ_{t-1} information to refer to the knowledge of all the likely series values as well as whatever that can be calculated using these values, such as squared observations and innovations. In theory, this can also incorporate data on the values of other linked time series and whatever else that would be significant for predicting that is accessible by time $t-1$.

A process ε_t is regarded as an *ARCH (q)* if the conditional distribution of ε_t given the available information ψ_{t-1} is:

$$\varepsilon_t / \psi_{t-1} \sim N(0, h_t) \quad (7)$$

$$h_t = \omega + \sum_{i=1}^{a} a_i \varepsilon_{t-i}^2 \quad (8)$$

With $w > 0,\ a_i \geq 0$ for all i and $\sum_{i=1}^{a} a_i < 1$

Equation (7) above shows that the conditional distribution of ε_t given φ_{t-1} is normally distributed as $N(0, h)$. For the information available ψ_{t-1} the subsequent observation ε_t is distributed as normal with a conditional mean $[\varepsilon_t / \psi_{t-1}] = 0$ and conditional variance of $Var[\varepsilon_t / \psi_{t-1}] = h_t$. These can be thought of as the mean and variance of ε_t calculated over all paths that agree with ψ_{t-1}. Equation (8) demonstrates how the information at hand determines the conditional variance h_t. It is important to note that the definition of h_t uses the squares of prior inventions. This ensures that h_t is positive, which it must be because it is a conditional variance, along with the assumptions that $\omega > 0$ and $a_i \geq 0$ are true.

RESULTS AND DISCUSSION

Table 1. Parameter estimates of the LC model

Age groups	a_x	b_x
0 – 1	-2.2445222	0.138443582
1-4	-3.8979299	0.174867719
5-9	-4.928847	0.06519156
10-14	-5.5600646	0.064015942
15 – 19	-5.1654801	0.052109864
20 – 24	-4.9314541	0.048287934
25 – 29	-4.8384513	0.045804646
30 – 34	-4.800877	0.044942453
35 – 39	-4.7222413	0.043885289
40 – 44	-4.6624831	0.041910333
45 – 49	-4.5648381	0.039741495
50 – 54	-4.2754396	0.03812534
55 – 59	-3.9441235	0.03611515
60 – 64	-3.446909	0.034482034
65 – 69	-2.9844628	0.031256629
70 – 74	-2.4762613	0.0274979
75 – 79	-1.9902172	0.023421014
80 – 84	-1.5194314	0.018904223
85 – 89	-1.1282181	0.014094418
90-94	-0.7887449	0.010414207
95+	-0.5928306	0.006488269

Table 2: Parameter Estimates For k_t Of The Lc Model

Years	K_t
2009	3.626629
2010	2.631595
2011	1.524175
2012	0.641461
2013	0.682196
2014	0.718416
2015	0.961365
2016	0.703628
2017	-0.83222
2018	-2.06571
2019	-3.50704
2020	-5.0845

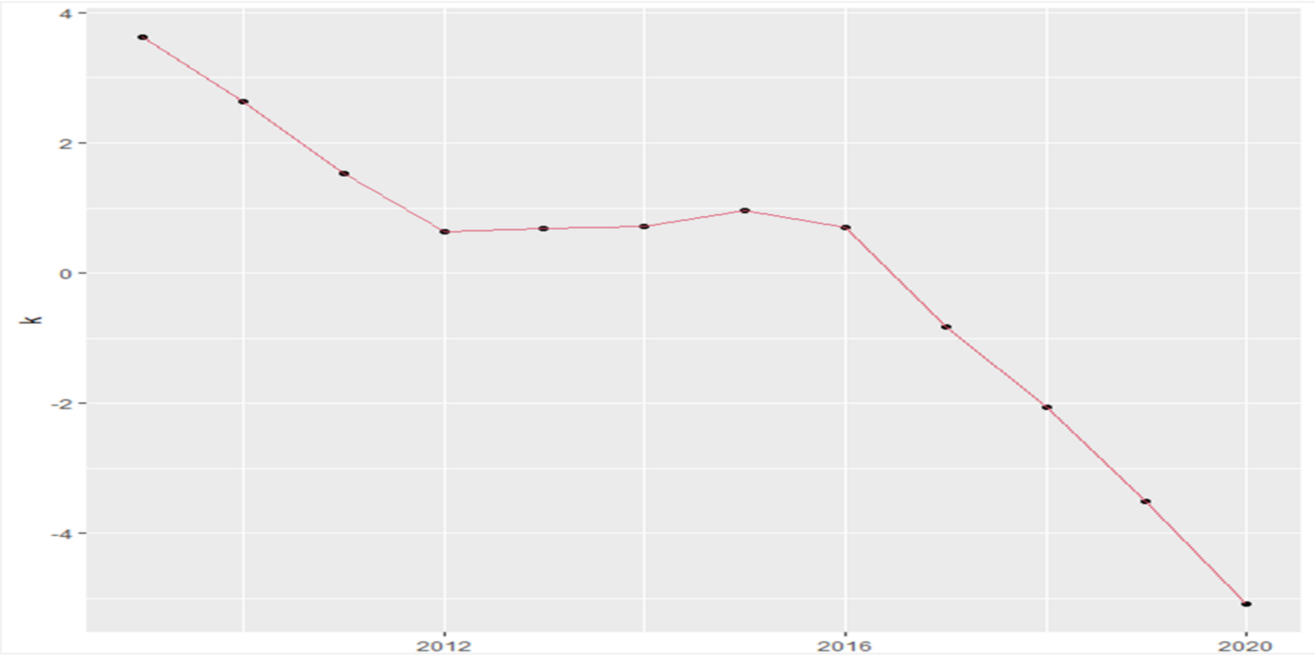

Figure 1. Overall Mortality Change over Time k_t from 2009 to 2020 for the LC model

The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used to select five (5) different ARIMA models in order to find the best model for modelling mortality in Nigeria.

Table 3: AIC and BIC for Suggested Arima Models

MODEL	AIC	BIC
ARIMA (1,1,1)	24.71	25.9065
ARIMA (1,1,2)	26.71	28.2995
ARIMA (0,1,0)	**18.33**	**18.6285**
ARIMA (0,1,2)	27.11	28.3001
ARIMA (2,1,0)	24.69	25.8798

Table 4: Forecasted values of k_t from 2021 to 2025 Using Arima Model

Year	2021	2022	2023	2024	2025
k_t	-6.661954	-8.239412	-9.816871	-11.394329	-12.971788

Testing for Arch effects

Langrage multiplier (LM) test was performed to determine the Presence and significance of ARCH effects prior to estimating ARCH. Before running the LM test the mean equation was estimated, which may be a regression of the variable on a constant and may include other variables. The estimated residuals are then saved, and their squares are calculated.

Arch langrage multiplier (LM) test TEST

Hypothesis

H_0: There is no ARCH effect

H_1: The series has ARCH effect

Level of significance: 0.05

Decision rule: Reject H_0 if $p-value \leq 0$

Table 5: Arch Langrage Multiplier (LM) Test

Chi-squared	60.884
Df	1
p-value	$6.053e^{-15}$

Conclusion: Since p-value = $6.053e^{-15} < \alpha = 0.05$, we reject H_0 and conclude that there is an ARCH effect.

Model Selection

To determine the most precise model for mortality in Nigeria, five (5) different ARCH models were fitted using the Akaike Information Criterion (AIC) as well as Bayesian Information Criterion (BIC).

Table 6: AIC and BIC for the Suggested Arch Models

MODEL	AIC	BIC
ARCH (1)	**-4.323858**	**-4.283222**
ARCH (2)	-4.313449	-4.259268
ARCH (3)	-4.311202	-4.243476
ARCH (5)	-4.301230	-4.206414
ARCH (6)	-4.257792	-4.214249

Based on the selection criterion of the above fitted models, it is clear that ARCH (1) has the least AIC, and BIC.

Model Comparison

The fitted values from the Lee-Carter and ARCH models were compared with the observed values. For the Lee-Carter and ARCH models, the RMSE (root mean square error) and MAPE (mean absolute percentage error) were estimated using the errors from fitted values.

Table 7: Selected Models from Lee-Carter and Arch

MODEL	LC model	**ARCH (1)**
RMSE	0.49968	**0.001732**
MAPE	37.34928	**14.89708**

Therefore having looked into the two selected models, ARCH (1) that is Autoregressive Conditional Heteroskedasticity one (1) has the least RMSE and MAPE and therefore it will be used to forecast the mortality rate of Nigeria.

CONCLUSION

In this study, it was found that the ARIMA model utilized by Lee and Carter was generally outperformed by the ARCH model in terms of fitting. ARCH (1) has the least RMSE and MAPE as compared to ARIMA (0,1,0) used by the LC model. Therefore Autoregressive Conditional Heteroskedasticity model was found to be the best model for modelling and predicting female mortality rates of Nigeria.

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