UMYU Scientifica, Vol. 2 NO. 3, September 2023, Pp 100 – 107
https://doi.org/10.56919/usci.2323.016
ISSN: 2955 – 1145 (print); 2955 – 1153 (online)
A periodical of the Faculty of Natural and Applied Sciences, UMYU, Katsina

ORIGINAL RESEARCH ARTICLE

New Generalized Odd Fréchet-G (NGOF-G) Family of Distribution with Statistical Properties and Applications.

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ABSTRACT
The distribution theory literature contains recent types of parametric distributional models that have been successfully used in the past and whose goodness of fit is sufficient only for certain datasets, suggesting further attention to accommodate a wider range of real-world datasets, for more adaptability, efficiency, and applications. This study aims to develop an extended Fréchet-G family of distributions and study their mathematical properties. The method of Alzaatreh is employed in developing a new lifetime continuous probability distribution called the new Generalized Odd Fréchet-G Family of Distribution. The developed distribution is flexible for studying positive real-life datasets. The statistical properties related to this family are obtained. The parameters of the family were estimated by using a technique of maximum likelihood. A New Generalized Odd Fréchet-Weibull model is introduced. This distribution was fitted with a set of lifetime data. A Monte Carlo simulation is applied to test the consistency of the estimated parameters of this distribution in terms of their bias and mean squared error with a comparison of M.L.E and the maximum product spacing (MPS). The findings of the Monte Carlo simulation show that the M.L.E method is the best technique for estimating the parameter of New Generalized Odd Frechet-Weibull distribution than the M.PS method. The findings of the application on the data set produce a higher flexibility than some of the competing distributions. In general, our new distributions serve as a viable alternative to other distributions available in the literature for modelling positive data.

ARTICLE HISTORY
Received August 03, 2023.
Accepted September 30, 2023.
Published September 30, 2023.

KEYWORDS
New Odd Fréchet-G Family, Moments, Hazard functions, Maximum Likelihood, Monte Carlo Simulations.

INTRODUCTION
René Fréchet constructed the Fréchet distribution as the maximum value distribution and gave it his name Fisher *et al.* (1928) and Gumbel (1958). As a result of experimental research from numerous disciplines, the Fréchet distribution (FD) is frequently utilized and explored for survival analysis and reliability investigation. The engineering, social, physical, environmental, and life sciences are all fields in which this continuous probability distribution is applicable, Deepshikha *et al.* (2021). In their observations, researchers, such as de Gusmão *et al.* (2011), and Khan *et al.* (2008) noted that the Fréchet distribution is essentially a type II extreme value distribution. The novelty of developing a generalized form of probability distribution drew the attention of academicians and devoted statisticians to the flexibility possessions of the generalized distributions. The generalized extreme value distribution (GEVD) is the result of the Weibull, Gumbel, and Fréchet distribution (GWF) being hybridized. Examples of generalized distribution include the Beta-G, the Gamma-G type I, and the Logistic-G, respectively developed by Eugene *et al.* (2002), Zografos *et al.* (2005) and Torabi *et al.* (2010). Mc-G developed by Alexander *et al.* (2012), Gamma-G type-3 by Torabi and Montazeri (2012), the Exponentiated-G family by Gupta *et al.* (1998), the Logistic-X family by Tahir *et al.* (2016), the Exponentiated-G class of distribution by Cordeiro *et al.* (2013), Odd Log-logistic by Cordeiro *et al.* (2017) and others like New Weibull-X by Ahmad *et al.* (2018). The T-X family of distributions was first published by Alzaatreh *et al.* (2013), and the exponentiated T-X family of distributions by Alzaghal *et al.* (2013). But Tahir *et al.* (2015) concentrated on the Poisson-X family, whereas Bourguignon *et al.* (2014) presented the Weibull-G family, the Beta Weibull-G by Yousof *et al.* (2017), the Generalized Odd Generalized Exponential-G by Alizadeh *et al.* (2017), the Jamal Weibull-X by Jamal and Nasir (2019), the One Parameter Generalized Odd Fréchet family by Marganpoor *et al.* (2020), the Rayleigh-EOG-X Family by Yahaya and Doguwa (2021), the New Weibull-Odd Fréchet-G Family by Usman *et al.* *(2021),* the

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How to cite: Abubakar Sadiq, I., Doguwa, S. I., Yahaya, A., & Garba, J. (2023). New Generalized Odd Fréchet-G (NGOF-G) Family of Distribution with Statistical Properties and Applications. *UMYU Scientifica*, 2(3), 100 – 107. https://doi.org/10.56919/usci.2323.016

Weibull–Odd Frechet-G Family by Usman *et al.* (2021), the Inverse Lomax-G by Falgore and Doguwa (2020), the One Parameter Odd Fréchet family by Ulhaq and Elgarhy (2018), Gamma-G type-2 by Torabi and Montazeri (2012), Nasir Weibull-G by Jamal and Nasir (2019), Jamal Logistic-X by Jamal and Nasir (2019), etcetera. However, the work of Ul-Haq and Elgarhy (2018), and Marganpoor *et al.* (2020) used one parameter Frechet family of distribution to describe and model datasets of extreme events. The recent occurrences of extreme events became global issues resulting in hazards to the government administration, lives, and properties. The one-parameter Frechet family appears sub-optimal and may not perform as its two-parameter version which will provide greater flexibility. Therefore, this work intends to develop and explore the full potential of two parameters Frechet's family of distribution for greater flexibility and applicability to extreme events. The two-parameter Fréchet distribution has different parameterizations, and we shall use the one defined by Ahmed *et al.* (2016), a random variable T is said to have a Fréchet distribution with scale parameter α and shape parameter β if its probability density function (pdf) and cumulative distribution function (CDF) are given as,

$$y(t\ ;\ \alpha\ ,\beta) = \beta\alpha^{\beta}t^{-(\beta+1)}\, exp\left\{-\left(\frac{\alpha}{t}\right)^{\beta}\right\};\ 0<t<\infty, \alpha,\beta>0 \quad (1)$$

$$Y(t\ ;\ \alpha\ ,\ \beta) = exp\left\{-\left(\frac{\alpha}{t}\right)^{\beta}\right\}\ ;\ \ 0<t<\infty\ , \alpha,\beta>0 \quad (2)$$

The Weibull distribution has different parameterizations, and we shall use the one defined by Usman *et al.* (2021). A random variable X is said to have Weibull distribution with scale parameter ϕ and shape parameter ω if its pdf and CDF are given as,

$$f(x\ ;\ \phi\ ,\ \omega) = \phi\omega x^{\omega-1}\, exp\{-(\phi x^{\omega})\}\ ;\ \ 0<x<\infty\ , \phi,\omega>0 \quad (3)$$

$$F(x\ ;\ \phi\ ,\ \omega) = 1 - exp\{-(\phi x^{\omega})\}\ ;\quad 0<x<\infty\ ,\ \phi,\omega>0 \quad (4)$$

NEW GENERALIZED ODD FRECHET-G FAMILY OF DISTRIBUTION

We proposed a new family of distribution called the New Generalized Odd Fréchet-G (NGOF-G) family by integrating the density function presented in equation (1) to obtain the CDF given by

$$F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\} \quad (5)$$

where $\alpha>0$ is the scale parameter, $\beta>0,\ \gamma>0$, is the shape parameter $F_{cdf}(x;\xi)$ is the CDF and ξ the parameters' vector of the baseline distribution. The corresponding pdf of the CDF in equation (5) is given by

$$f_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \beta\gamma\alpha^{\beta} f_{pdf}(x;\xi)F_{cdf}^{-(\gamma+1)}(x;\xi)\left[F_{cdf}^{-\gamma}(x;\xi)-1\right]^{\beta-1} exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\} \quad (6)$$

where $f_{pdf}(x;\xi)$ is the pdf and ξ is the parameters' vector of the baseline distribution. Hereafter, a random variable X with density function and distribution function in equations (6) and (5) is denoted by

$$X\sim NGOF-G\ (\alpha,\beta,\gamma,\xi).$$

Survival and Hazard Rate Function of the NGOF-G Family

The survival function, hazard function, and cumulative hazard function are respectively given as,

$$S_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = 1 - exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\} \quad (7)$$

$$h_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \frac{\beta\gamma\alpha^{\beta} f_{pdf}(x;\xi)F_{cdf}^{-(\gamma+1)}(x;\xi)\left[F_{cdf}^{-\gamma}(x;\xi)-1\right]^{\beta-1} exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\}}{1-exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\}} \quad (8)$$

$$H_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = -log\left(1 - exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\}\right) \quad (9)$$

Quantile Function of NGOF-G Family

The quantile function of the NGOF-G family for a simulation through inverting the CDF in equation (5). Supposed that $u[0,\ 1]$ follows a rectangular distribution, hence, easily derived by the following expressions:

$$x = F_{cdf}^{-\gamma}(x;\xi)\left[\frac{\alpha}{\left(\alpha+(-log(u))^{\frac{1}{\beta}}\right)}\right] \quad (10)$$

where $F_{cdf}^{-\gamma}(x;\xi)$ is the inverted CDF for every future baseline F and ξ is the parameters' vector

Suitable Expansion for the PDF and CDF of the NGOF-G Family

Here, we consider the individual terms in the given CDF and pdf of the NGOF-G family presented in equations (5) and (6). Some standard mathematical expansion will be operated upon them, comprising the generalized binomial expansion for negative and positive power, the power series expansion and so on. For instance,

$$\left[e^{-ax} = \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}(ax)^i and\ (1-x)^{ab} = \sum_{j=0}^{\infty}(-1)^j\binom{ab}{j}(x)^j\ \right],$$

however, equation (5) is the same as,

$$F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = exp\left\{-\left(\alpha\left(F_{cdf}^{-\gamma}(x;\xi)-1\right)\right)^{\beta}\right\}$$

$$= e^{-\left[\frac{\alpha\left(1-F_{cdf}^{\gamma}(x;\xi)\right)}{F_{cdf}^{\gamma}(x;\xi)}\right]^{\beta}}$$

$$= \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left[\alpha\left(\frac{1-F_{cdf}^{\gamma}(x;\xi)}{F_{cdf}^{\gamma}(x;\xi)}\right)\right]^{i\beta}$$

$$= \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\alpha^{i\beta}\left(1-F_{cdf}^{\gamma}(x;\xi)\right)^{i\beta}F_{cdf}^{-\gamma i\beta}(x;\xi)$$

$$= \sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\alpha^{i\beta}\sum_{j=0}^{\infty}(-1)^j\binom{ij\beta}{j}\left(F_{cdf}^{\gamma i\beta}(x;\xi)\right)^j F_{cdf}^{-\gamma i\beta}(x;\xi)$$

$$= \sum_{i,j=0}^{\infty}\frac{(-1)^{i+j}}{i!}\alpha^{i\beta}\binom{ij\beta}{j}F_{cdf}^{\gamma i\beta(j-1)}(x;\xi) \qquad (11)$$

Therefore, equation (11) reduces to,

$$F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \sum_{i,j=0}^{\infty}Q_{i,j}F_{cdf}^{\gamma i\beta(j-1)}(x;\xi) \qquad (12)$$

where $Q_{i,j} = \frac{(-1)^{i+j}}{i!}\alpha^{i\beta}\binom{ij\beta}{j}$

Differentiating equation (12) w.r.t. x we have the corresponding pdf as:

$$f_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \sum_{i,j=0}^{\infty}Q_{i,j}\,\gamma i\beta(j-1)f_{pdf}(x;\xi)F_{cdf}^{\gamma i\beta(j-1)-1}(x;\xi) \qquad (13)$$

Further simplification of equation (12) is as,

$$F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \sum_{k=0}^{\infty}b_kA_k(x) \qquad (14)$$

where $b_k = \sum_{i,j=0}^{\infty}Q_{i,j}$ *and* $A_k(x) = F_{cdf}^{\gamma i\beta(j-1)}(x;\xi)$

Differentiate equation (14) w.r.t. x we obtained the corresponding pdf as:

$$f_{NGOF-G}(x;\alpha,\beta,\gamma,\xi) = \sum_{k=0}^{\infty}b_ka_k(x) \qquad (15)$$

where $a_k(x) = kf_{pdf}(x;\xi)F_{cdf}^{k-1}(x;\xi)$

Special Sub-Model of the NGOF-G Family

Setting equations (3) and (4) into (5), (6), (7), (8) and (9), then $f(x)$, $F(x)$, $S(x)$, $h(x)$, and $H(x)$ will be for the NGOF-Weibull distribution.

Graph of the Special Sub-Model of the NGOF-G Family

The plot of the density function and the hazard function of the NGOF-Weibull distributions are:

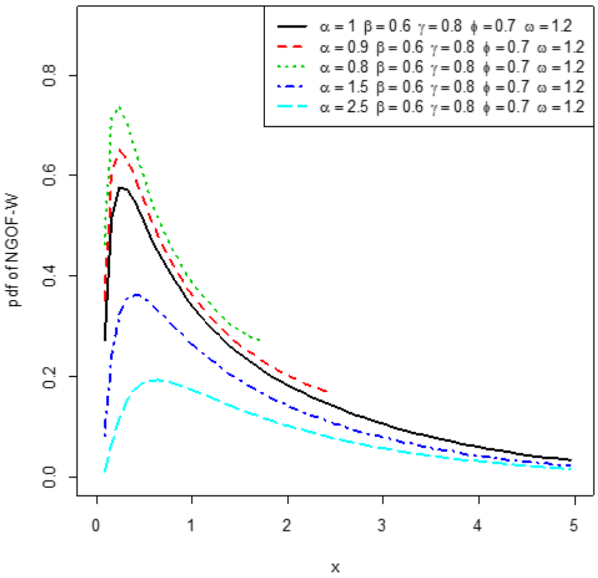
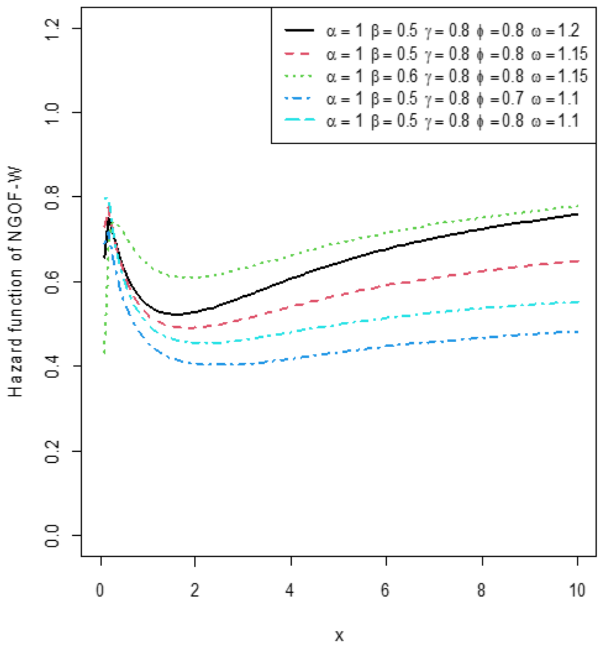


Figure 1: *pdf Plot of the New Generalized Odd Fréchet-Weibull Distribution*

Figure 1 displays the shapes and the behaviour of the density function of the New Generalized Odd Frechet-Weibull model at different parameter values. It is also seen how different parameters interact with one another. For instance, in the pdf plots, if the parameters are assumed to equal values, the distribution is symmetrical, whereas it is more positively skewed the greater the difference between the parameters' values and the bell shape, the not greater the difference between the parameters' values.



Figures 2: HF Plot of the New Generalized Odd Fréchet-Weibull Distribution

Figure 2 displays the shapes and the behaviour of the hazard functions of the New Odd Frechet-Weibull distribution at different parameter values. The graph shows modified unimodal and modal shapes of hazard rates at different values of the parameters.

Moments of the NGOF-G Family

The role of moments in application to statistics is clear, and the most essential characteristics of a probability model can be examined using moments. Evaluation in statistical inference is necessary, the most vital properties of the distributions were derived using the moments. The rth ordinary moment of a random variable X that follows the New Generalized Odd Frechet-G (NGOF-G) family is given as:

$$\mu_r^{/} = E(X^r)$$

$$= \int_0^{\infty} x^r f_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)dx$$

$$= \int_0^{\infty} x^r \sum_{k=0}^{\infty} b_k a_k(x)\,dx$$

$$= \sum_{k=0}^{\infty} b_k \int_0^{\infty} x^r a_k(x)\,dx$$

$$= \sum_{k=0}^{\infty} b_k E[Z_k^r] \quad (16)$$

where $E[Z_k^r] = \int_0^{\infty} x^r k f_{pdf}(x;\xi) F_{cdf}^{k-1}(x;\xi)\,dx$

Moment-Generating Function of the NGOF-G Family

The moment-generating function of a random variable X that follows the New Generalized Odd Frechet-G (NGOF-G) family is given as:

$$M_X^{NGOF-G}(t) = E(e^{tx})$$

$$= \int_0^{\infty} e^{tx} f_{NGOF-G}(x;\alpha,\beta,\gamma\xi)dx$$

$$= \int_0^{\infty} e^{tx} \sum_{k=0}^{\infty} b_k a_k(x)\,dx$$

$$= \sum_{k=0}^{\infty} b_k \int_0^{\infty} e^{tx} a_k(x)\,dx$$

$$= \sum_{k=0}^{\infty} b_k E[e^{tZ_k}] \quad (17)$$

where $E[e^{tZ_k}] = \int_0^{\infty} e^{tx} k f_{pdf}(x;\xi) F_{cdf}^{k-1}(x;\xi)\,dx$

Entropies of the NGOF-G Family

The entropy of any random variable X is a measure of indecisiveness, variability, and details properties of the probable results of the variable. The entropy of the NGOF-G family is defined mathematically as:

$$I_R(\varpi) = \frac{1}{1-\varpi} log\left(\int_0^{\infty} f^{\varpi}{}_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)\,dx\right)$$

$$= \frac{1}{1-\varpi} log\left(\int_0^{\infty} \left(\sum_{k=0}^{\infty} b_k a_k(x)\right)^{\varpi} dx\right) \quad (18)$$

where $\varpi > 0$and $\varpi \neq 1$

The nth entropy is defined by

$$I_{nth}(\varpi) = \frac{1}{\varpi - 1} log\left(1 - \int_0^{\infty} f^{\varpi}{}_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)\,dx\right)$$

$$= \frac{1}{1-\varpi} log\left(1 - \int_0^{\infty} \left(\sum_{k=0}^{\infty} b_k a_k(x)\right)^{\varpi} dx\right) \quad (19)$$

Order Statistics of the NGOF-G Family

Suppose $X_1, X_2, X_3, \ldots, X_n$ is a random sample from the NGOF-G distribution and $X_{i:n}$ represent the ith order statistic, then

$$f_{i:n}(x;\alpha,\beta,\xi) = \frac{n!}{[(i-1)!\,(n-i)!]}[f_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)] \times [F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)]^{i-1}[1 - F_{NGOF-G}(x;\alpha,\beta,\gamma,\xi)]^{n-i}$$

$$= \frac{n!}{[(i-1)!\,(n-i)!]}\left[\sum_{k=0}^{\infty} b_k a_k(x)\right]\left[\sum_{k=0}^{\infty} b_k A_k(x)\right]^{i-1}\left[1 - \sum_{k=0}^{\infty} b_k A_k(x)\right] \quad (20)$$

Estimation of Parameters of the NGOF-G Family

Suppose that $x_1, x_2, x_3, \ldots, x_n$ are the observed values from the proposed NGOF-G family with parameters $\alpha, \beta, \gamma,$. Suppose that $\Phi = [\alpha, \beta, \gamma]^T$ is the $[m \times 1]$ vector of the parameter. The log-likelihood function Φ is expressed by

$$\ell_n = \ell_n(\Phi) = n\,log(\beta) + n\,log(\gamma) + n\beta\,log(\alpha) + \sum_{i=1}^{n} log[f_{pdf}(x;\xi)] - (\gamma+1)\sum_{i=1}^{n} log[F_{cdf}(x;\xi)] + (\beta-1)\sum_{i=1}^{n} log[F_{cdf}^{-\gamma}(x;\xi) - 1] - \sum_{i=1}^{n}[\alpha(F_{cdf}^{-\gamma}(x;\xi)-1)]^{\beta} \quad (21)$$

Taking the partial derivative of equation (21) w.r.t. the parameters $(\alpha;\ \beta;\ \gamma)$are respectively given as:

$$\frac{\partial \ell_n(\Phi)}{\partial \alpha} = \frac{n\beta}{\alpha} - \sum_{i=1}^{n}[(F_{cdf}^{-\gamma}(x;\xi)-1)]^{\beta} \quad (22)$$

$$\frac{\partial \ell_n(\Phi)}{\partial \beta} = \frac{n}{\beta} + n\,ln(\alpha) + \sum_{i=1}^{n} ln\left[F_{cdf}^{-\gamma}(x;\xi) - 1\right] - \sum_{i=1}^{n} \left[\alpha\left(F_{cdf}^{-\gamma}(x;\xi) - 1\right)\right]^{\beta} ln\left[\alpha\left(F_{cdf}^{-\gamma}(x;\xi) - 1\right)\right] \quad (23)$$

$$\frac{\partial \ell_n(\Phi)}{\partial \gamma} = \frac{n}{\gamma} + n\,ln(\alpha) + \sum_{i=1}^{n} ln\left[F_{cdf}^{-\gamma}(x;\xi) - 1\right] + (\beta - 1)\sum_{i=1}^{n} \frac{f_{pdf}(x;\xi)F_{cdf}^{-(\gamma+1)}(x;\xi)}{\left(F_{cdf}^{-\gamma}(x;\xi) - 1\right)} + \sum_{i=1}^{n} f_{pdf}(x;\xi)F_{cdf}^{-(\gamma+1)}(x;\xi)\left[\alpha\left(F_{cdf}^{-\gamma}(x;\xi) - 1\right)\right]^{\beta} \quad (24)$$

The MLEs of the parameters $(\alpha;\ \beta;\ \gamma)$, says $(\hat{\alpha};\ \hat{\beta};\ \hat{\gamma})$are the simultaneous solution of equations (22), (23), and (24) equating them to zero, i.e $\frac{\partial \ell_n(\Phi)}{\partial \alpha} = 0$; $\frac{\partial \ell_n(\Phi)}{\partial \beta} = 0$; $\frac{\partial \ell_n(\Phi)}{\partial \gamma} = 0$. These equations are intractable and can only be solved using a numerical iterative method.

RESULT OF THE MONTE CARLO SIMULATION

The vast class of computational algorithms known as "Monte Carlo simulations" uses replicated random sampling to produce numerical results. The basic idea is to employ randomness to address problems that could be theoretically deterministic.

M.L.E and M.P.S Techniques

To evaluate the consistency of the new family's parameters, the simulation study was piloted using the Monte Carlo Simulation technique by computing the bias, variance and mean square error of the estimated parameters from the maximum likelihood estimates and the maximum product spacing estimate. The Simulated data is generated using the quantile function in equation (10) and the likelihood function in equation (21) for different sample sizes n = 50, 100, 150 with replicate 200 times each. For the NOGF-Weibull distribution parameter values are $(\alpha, \beta, \gamma, \phi, \omega) = (5.0, 1.0, 2.5, 2.0, 3.0)$.

Table 1: Results of the simulated data from the NGOF-Weibull Distribution.

Sample Sizes	Parameters (Actual Values)	M.L.E. Techniques			M.P.S. Techniques		
		Estimates	Bias	RMSE	Estimates	Bias	RMSE
50	α (5.0)	5.0064	0.0064	0.1763	4.1405	-0.8595	1.0245
	β (1.0)	0.9719	-0.0281	0.1429	0.2224	-0.7776	0.7782
	γ (2.5)	2.6584	0.1584	0.3203	3.5509	1.0509	1.2231
	ϕ (2.0)	1.9829	-0.0171	0.1583	0.5131	-1.4869	1.4908
	ω (3.0)	2.9647	-0.0353	0.2416	4.4028	1.4028	1.4928
100	α (5.0)	5.0006	0.0006	0.0200	4.1942	-0.8058	0.9490
	β (1.0)	0.9992	-0.0008	0.0263	0.2274	-0.7726	0.7730
	γ (2.5)	2.5672	0.0672	0.1722	3.4259	0.9259	1.0724
	ϕ (2.0)	1.9987	-0.0013	0.0427	0.5050	-1.4950	1.4982
	ω (3.0)	3.0004	0.0004	0.0139	4.3642	1.3642	1.4462
150	α (5.0)	5.0000	0.0000	0.0000	4.2464	-0.7536	0.9054
	β (1.0)	1.0000	0.0000	0.0000	0.2285	-0.7715	0.7719
	γ (2.5)	2.5387	0.0387	0.1192	3.3680	0.8680	1.0000
	ϕ (2.0)	2.0000	0.0000	0.0000	0.5002	-1.4998	1.5030
	ω (3.0)	3.0000	0.0000	0.0000	4.3476	1.3476	1.4255

Table 1 presents the results obtained from the Monte Carlo Simulation study. The results indicated that the bias and root mean square error decrease toward zero with an increase in sample size. However, the actual value of the parameters and the estimated values are almost equal at different sample sizes and iterative levels for the M.L.E technique. This proves the consistency of the MLE parameter estimates. For the M.PS technique, the actual value of the parameters and the estimated values are almost not equal at different sample sizes and iterative levels. This proves the least consistency of the M.P.S parameter estimates. The result also means that the M.L.E technique is the best technique for estimating the parameter of New Generalized Odd Frechet-Weibull distribution than the M.PS technique.

APPLICATIONS

Here we used some existing real-life data sets to assess the flexibility of our developed family using Weibull distribution as a baseline.

Some of the Competing Models

Ul-Haq and Elgarhy (2018) introduce a random variable X that is said to follow the Odd Frechet-Weibull distribution if its pdf and CDF are given as

$$f(x;\beta,\phi,\omega)=\beta g(x,\xi)\left((1-exp\{-(\phi x^{\omega})\})\right)^{-(\beta+1)}\left(1-(1-exp\{-(\phi x^{\omega})\})\right)^{\beta-1}\times exp\left\{-\left(\frac{1-(1-exp\{-(\phi x^{\omega})\})}{(1-exp\{-(\phi x^{\omega})\})}\right)^{\beta}\right\};\quad \forall x,\beta,\phi,\omega>0 \tag{25}$$

$$F(x;\beta,\phi,\omega)=exp\left\{-\left(\frac{1-(1-exp\{-(\phi x^{\omega})\})}{(1-exp\{-(\phi x^{\omega})\})}\right)^{\beta}\right\};\quad \forall x,\beta,\phi,\omega>0 \tag{26}$$

Marganpoor *et al.* (2020) introduced a random variable X that is said to follow the Generalized Odd Frechet-Weibull distribution if its pdf and CDF are given as

$$f(x;\alpha,\beta,\phi,\omega)=\alpha\beta(\phi\omega x^{\omega-1}exp\{-(\phi x^{\omega})\})((1-exp\{-(\phi x^{\omega})\})^{-\alpha-1})((1-exp\{-(\phi x^{\omega})\})^{-\alpha}-1)^{\beta-1}exp\{-(G(x,\phi)^{-\alpha}-1)^{\beta}\};\quad \forall x,\alpha,\beta,\phi,\omega>0 \tag{27}$$

$$F(x;\alpha,\beta,\phi,\omega)=exp\{-((1-exp\{-(\phi x^{\omega})\})^{-\alpha}-1)^{\beta}\};\quad \forall x,\alpha,\beta,\phi,\omega>0 \tag{28}$$

Application on Maximum Annual Flood Discharges Data

The data set was originally reported by Montfort (1970) which represents the Maximum Annual Flood Discharges of North Saskatchewan in units of 1000 cubic feet per second, of the North Saskatchewan River at Edmonton, for 47 years. The data are: 19.885, 20.940, 21.820, 23.700, 24.888, 25.460, 25.760, 26.720, 27.500, 28.100, 28.600, 30.200, 30.380, 31.500, 32.600, 32.680, 34.400, 35.347, 35.700, 38.100, 39.020, 39.200, 40.000, 40.400, 40.400, 42.250, 44.020, 44.730, 44.900, 46.300, 50.330, 51.442, 57.220, 58.700, 58.800, 61.200, 61.740, 65.440, 65.597, 66.000, 74.100, 75.800, 84.100, 106.600, 109.700, 121.970, 121.970, 185.560.

Table 2: Parameters Estimates and Goodness of Fit Measures for Maximum Annual Flood Discharges Data

Model	Parameter Estimates and Goodness of Fit						
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\phi}$	$\hat{\omega}$	LL	AIC
NGOFWD	0.3144	2.1499	0.6957	0.0065	0.6619	179.2229	**368.4458**
GOFWD	2.0973	4.9876	0.5524	0.2328	-	215.0636	438.1276
OFWD	1.2325	8.3788	1.4295	-	-	1918.734	3831.468
EtWD	2.3463	1.8221	7.2015	-	-	266.6662	489.6832
WD	0.0007	1.7724	-	-	-	225.7065	455.4135
FD	35.246	2.4462	-	-	-	215.1136	434.2272

Table 2 provides the parameter estimates and goodness of fit measures for the New Generalized Odd Frechet-Weibull distribution with other competing models using the maximum annual flood discharges dataset. Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQIC) are the performance metrics. A distribution with the lowest information or performance metrics is regarded as the best in terms of goodness of fit. The new generalized odd Frechet-Weibull distribution is the best model that outperforms other competitors based on the data set.

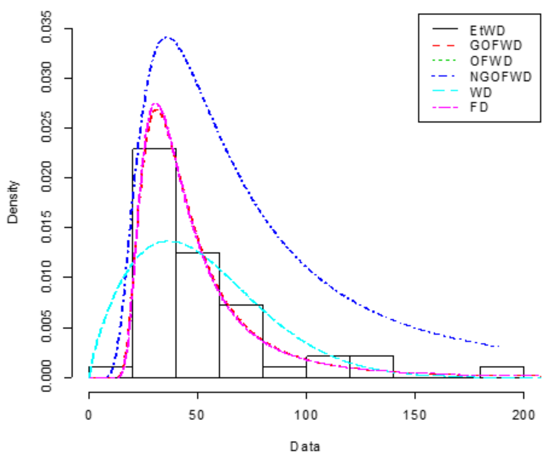


Figure 3: Histogram Plots of the Distribution of Maximum Annual Flood Discharges Data

CONCLUSION

In this paper, we developed the New Generalized Odd Frechet-G (NGOF-G) family of distributions. The statistical properties of this family comprising the survival function, hazard function, cumulative hazard function, moments, moment generating function, entropies, order statistics, and MLE are obtained. We further plot the pdf and the hazard rate function to assess the shapes and behaviour of the models at different parameter values. Simulation studies were carried out for the assessment of the consistents of MLE and MPS of the parameters. We present the application of the NGOF-W distribution to the data representing Maximum Annual Flood Discharges employing Weibull as the baseline distribution. The analysis indicated that the NGOFW is the "best fit" model for the Maximum Annual Flood discharge data.

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