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**ORIGINAL RESEARCH ARTICLE**

# Commutativity of some Prime Nearrings using Left-Sided outer (σ, τ)-n-Derivation

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**ABSTRACT**
In the field of mathematics, pure mathematics is very important as it gives rise to the basis for the formation of all applicable mathematical concepts in solving real-life problems. Algebra is such an integral part of pure mathematics. It consists of the Ring theory. It has been discovered that there exists some structures similar to rings with little deformity and they are called NEARRINGS. These structures do not commute mostly as they fail to satisfy distributive law. To ascertain the commutativity of nearrings, we need derivation(s). Because several papers had been presented dealing with left-nearrings, this paper aimed to consider right-nearrings which has not been done before in that respect, to the best of our knowledge. Some methods dealing with left-nearrings have been studied and modified in this work, and new derivation has been introduced to take care of right-nearrings. For Let $n$ be a positive integer, $N$ be a right nearring, and automorphisms $\sigma, \tau: N \to N$, in this study, the concept of left-sided outer (σ, τ)-$n$-derivations on the nearrings is introduced, and several features are examined to show commutativity of prime right nearring using the derivation. The results obtained in this paper show that the right-nearrings are commutative when subjected to derivation.

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## INTRODUCTION

An algebraic structure or a triple $(N, +, \cdot)$ containing a non-null set $N$ together with binary operations (addition '+' as well as multiplication '$\cdot$') that meet all the axioms of rings except for commutativity of addition and just one distributive law holds, is called a nearring. Precisely, right nearring is a triple $(N, +, \cdot)$ as such, (i) $(N, +)$ makes a group (ii) $(N, \cdot)$ is a semi-group (iii) $(s+t)r = sr + tr$, for all $r, s, t \in N$. See Clay (1992) and Pilz (1983) to learn more.

In this article, the right nearring is denoted by $N$ and $\tau$ are automorphisms of $N$. The nearrings' centre is indicated by $Z$ and is defined to be a collection $\{x \in N | xy = yx, \forall y \in N\}$. A right near- ring $N$ is called a prime nearring if $xNy = \{0\}$ then $x = 0$ or $y = 0$, for all $x, y \in N$. $N$ is semiprime nearring if; $xNx = \{0\}$ then $x = 0$ for all $x \in N$ and $N$ is commutative if $(N, \cdot)$ is commutative (that is $xy - yx = 0 \ \forall x, y \in N$). The symbol $[x, y]$ will be denoted by $xy - yx$ for $x, y \in N$ and is called Lie product (multiplicative commutator) of $x$ and $y$.

Prime nearrings: these are a special class of nearrings with the property that $xNy = \{0\}$ then $x = 0$ or $y = 0$, for all $x, y \in N$, where $N$ is the nearring of reference. These are very important, especially in the study of commutativity, as they enable us to use the primness property to test the commutativity stance of the structure. Some examples of prime nearrings include: [1] Let $(N, +)$ be a group; we define an operation $*$ on $N$ by $x * y = x$ for all $x, y \in N$, it is observed that $(N, +, *)$ is a right prime nearing. [2] Let $(N, +)$ be a Klen four group. Consider the following table;

| $*$ | 0 | r | s | t |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| r | 0 | 0 | r | r |
| s | 0 | r | s | s |
| t | 0 | r | t | t |

Clearly $(N, +, *)$ is prime right nearing, since for $r, s \in N \Rightarrow r * N * s = \{0\}$only if $r = 0$ or $s = 0$.

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Ashraf and Siddeeque (2013) have come up with $(\sigma,\tau)$-$n$-derivation in nearrings and explored some conditions via $(\sigma,\tau)$-$n$-derivations on left prime nearrings. They defined on a given left nearring $N$ and distinct positive integer $n$, " an $n$-additive mapping $d: N \times N \times N \times \ldots \times N \to N$ is said to be $(\sigma,\tau)$-$n$-derivation of $N$ if there are functions $\sigma,\tau: N \to N$ such that the following hold for all $u_1,u_2,u_3,\ldots,u_n,u'_1,u'_2,u'_3,\ldots,u'_n \in N$;

$$d(u_1u'_1,u_2,u_3,\ldots,u_n) = d(u_1,u_2,u_3,\ldots,u_n)\sigma(u'_1) + \tau(u_1)d(u'_1,u_2,u_3,\ldots,u_n)$$

$$d(u_1,u_2u'_2,u_3,\ldots,u_n) = d(u_1,u_2,u_3,\ldots,u_n)\sigma(u'_2) + \tau(u_2)d(u_1,u'_2,u_3,\ldots,u_n)$$

$$d(u_1,u_2,u_3u'_3,\ldots,u_n) = d(u_1,u_2,u_3,\ldots,u_n)\sigma(u'_3) + \tau(u_3)d(u_1,u_2,u'_3,\ldots,u_n)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$d(u_1,u_2,u_3,\ldots,u_nu'_n) = d(u_1,u_2,u_3,\ldots,u_n)\sigma(u'_n) + \tau(u_n)d(u_1,u_2,u_3,\ldots,u'_n).$$

**Remark**: Once $d$ is permuting, all the above are equivalents."

They studied certain properties via the derivation and proved some results, such as;

Result#(1) Given a prime nearring $N$ and $D \neq 0$ be a $(\sigma,\tau)$-$n$-derivation on $N$. If $D(N,\ldots,N) \subseteq Z$, then $(N,\ \cdot)$ is commutative.

Result#(2) Assume a prime nearring $N$ and $D_1 \neq 0$ and $D_2 \neq 0$ be $(\sigma,\tau)$-$n$-derivations of $N$. If $[D_1,D_2] = \{0\}$, then $(N,+)$ is abelian.

Aroonruviwat and Leerawat (2021a) and (2021b), motivated by these results and some results in the above paper, defined Outer $(\sigma,\tau)$-$n$-derivation on a left nearring as follows; "Assume $N$ to be nearring and automorphisms $\sigma\,\&\,\tau$ be automorphisms from $N$ to itself. A map $d: N^n \to N$ is Outer $(\sigma,\tau)$-$n$-derivation once it is $n$-additive map where the following;

$$d(u_1u'_1,u_2,u_3,\ldots,u_n) = \sigma(u_1)d(u'_1,u_2,u_3,\ldots,u_n) + d(u_1,u_2,u_3,\ldots,u_n)\tau(u'_1)$$

$$d(u_1,u_2u'_2,u_3,\ldots,u_n) = \sigma(u_2)d(u_1,u'_2,u_3,\ldots,u_n) + d(u_1,u_2,u_3,\ldots,u_n)\tau(u'_2)$$

$$d(u_1,u_2,u_3u'_3,\ldots,u_n) = \sigma(u_3)d(u_1,u_2,u'_3,\ldots,u_n) + d(u_1,u_2,u_3,\ldots,u_n)\tau(u'_3)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$d(u_1,u_2,u_3,\ldots,u_nu'_n) = \sigma(u_n)d(u_1,u_2,u_3,\ldots,u'_n) + d(u_1,u_2,u_3,\ldots,u_n)\tau(u'_n)$$

Holds for all $u_1,u_2,u_3,\ldots,u_n,u'_1,u'_2,u'_3,\ldots,u'_n \in N$.

In addition, they studied certain properties, proved the commutativity of prime nearrings, and came up with some results.

Result#(3) For a prime nearring $N$ and non-zero $D$, outer $(\sigma,\tau)$-$n$-derivation of $N$. If $D(N,\ldots,N) \subseteq Z$, then $(N,\ \cdot\ )$ commute.

Result#(4) Let $N$ be a prime nearring and $D_1$ and $D_2$be a non-zero $(\sigma,\tau)$-$n$-derivations of $N$. If $[D_1(x_1,x_2,x_3,\ldots,x_n),D_2(y_1,y_2,y_3,\ldots,y_n)] = 0$, for all $x_1,x_2,x_3,\ldots,x_n,y_1,y_2,y_3,\ldots,y_n \in N$, then $x+y = y+x$ for all $x,y \in N$.

It is important we state the result, which was proven by Bell and Mason (1987)

Result# (5) Consider a prime nearring $N$ and $Z$ to be a multiplicative centre of the $N$. As $z \in Z - \{0\}$ for which; $z+z \in Z$, then $(N,+)$ is abelian.

In line with this investigation, it is observed that the previous researchers have done their work on left nearrings, this paper discussed right nearrings. To the best of our knowledge previous research were inclined to left nearrings. So our work is the first to dive into the nearrings in this respect. We will define the derivation and give examples and some theorems with proofs.

## METHODOLOGY

### Left-Sided Outer $(\sigma,\tau)$-$n$-Derivation

In this section, we define left-sided $(\sigma,\tau)$-$n$-derivation on $N$ and give examples.

Suppose $N$ is a nearring and $\sigma$ and $\tau$ are two automorphisms of $N$. A map $\delta: N^n \to N$ is said to be a left-sided outer $(\sigma,\tau)$-$n$-derivation on $N$ if $\delta$ is $n$-additive map such that the following;

$$d(u_1u_1', u_2, u_3, \dots, u_n) = \sigma(u_1)d(u_1', u_2, u_3, \dots, u_n) + \tau(u_1')d(u_1, u_2, u_3, \dots, u_n)$$

$$d(u_1, u_2u_2', u_3, \dots, u_n) = \sigma(u_2)d(u_1, u_2', u_3, \dots, u_n) + \tau(u_2')d(u_1, u_2, u_3, \dots, u_n)$$

$$d(u_1, u_2, u_3u_3', \dots, u_n) = \sigma(u_3)d(u_1, u_2, u_3', \dots, u_n) + \tau(u_3')d(u_1, u_2, u_3, \dots, u_n)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$d(u_1, u_2, u_3, \dots, u_nu_n') = \sigma(u_n)d(u_1, u_2, u_3, \dots, u_n') + \tau(u_n')d(u_1, u_2, u_3, \dots, u_n)$$

Hold for all $x_1, x_1', x_2, x_2', x_3, x_3', \dots, x_n, x_n' \in N$

The following example yields $\delta$ on $(N,+,\cdot)$ right nearring is left-sided outer $(\sigma,\tau)$-3-derivation.

**Example 1**; Let $N = \{0, r, s, t\}$ with the following tables;

**Table 1: Addition Table**

| + | 0 | r | s | t |
|---|---|---|---|---|
| 0 | 0 | r | s | t |
| r | r | 0 | t | s |
| s | s | t | 0 | r |
| t | t | s | r | 0 |

**Table 2: Multiplication Table**

| * | 0 | r | s | t |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| r | 0 | 0 | r | r |
| s | 0 | r | s | s |
| t | 0 | r | t | t |

Therefore, $(N,+,\cdot)$ is a right nearring.

Particularly, we define a map $\delta$ left-sided $(\sigma,\tau)$-3-derivation on $N$ by;

$$\delta(u,v,w) = \begin{cases} s, & u,v,w \notin \{0,r\} \\ 0, & otherwise \end{cases}.$$

Also $\sigma(r) = \sigma(0) = 0,\ \sigma(s) = s,\ \sigma(t) = t,\ \tau(u) = 0$ for all $u \epsilon N$

Now, $\delta(sr, s, t) = \delta(r, s, t) = 0$

Again, $\delta(sr,s,t) = \sigma(s)\delta(r,s,t) + \tau(r)\delta(s,s,t)$

$$= s\delta(r,s,t) + (0)\delta(s,s,t)$$

$$= s(0) + (0)s = 0$$

Hence, $\delta$ is the left-sided outer $(\sigma,\tau)$-3-derivation on $N$.

In this line of observations, we define a map $\delta$ by;

$$\delta(\propto_1, \dots, \propto_n) = \begin{cases} \propto_2, & \propto_1, \dots, \propto_n \notin \{0, r\} \\ 0, & otherwise \end{cases}$$

$\delta$ is a left-sided outer $(\sigma,\tau)$-$n$-derivation on $N$.

**Example 2;** Let $R$=$(Z,+,\cdot)$ be a ring. Let $P(R) = \{ax + b \mid a, b \in R\}$, define operations addition " $+$ " and substitution " $\circ$ " by:

(a) Addition " $+$ " defined on $P(R)$ is $\forall P_1, P_2 \in N$, where $P_1 = a_1x + b_1$ and $P_2 = a_2x + b_2$ implies that $P_1 + P_2 = a_1x + b_1 + a_2x + b_2 = (a_1 + a_2)x + (b_1 + b_2) = a_3x + b_3 = P_3 \in P(R)$ and $a_3, b_3 \in R$.

(b) Substitution " $\circ$ " on $P(R)$ is $\forall P_1, P_2 \in N$ implies that $P_1(P_2) = P_1(a_2x + b_2) = a_1(a_2x + b_2) + b_1 = a_1a_2x + a_1b_2 + b_1 = a_4x + b_4 = P_4 \in P(R)$ and $a_4, b_4 \in R$.

Then $N = (P(R), +, \circ)$ is a right prime-nearring. Please observe that it is non-commutative. Define a left-sided outer $(\sigma,\tau)$ derivation on $N$ by a map $\delta: N \to N$ by $\delta(P) = \overline{P}$, where $P = ax + b$ and $\overline{P} = ax$. Also $\sigma(P) = \overline{P}$ and $\tau(P) = 0$ for all $P \in N$.

Consider $P_1, P_2 \in N$, then $P_1 \circ P_2 \neq P_2 \circ P_1$.

$$P_1 \circ P_2 = P_1(P_2) = a_1P_2 + b_1 = a_1(a_2x + b_2) + b_1$$
$$= a_1a_2x + a_1b_2 + b_1$$

Again;

$$P_2 \circ P_1 = P_2(P_1) = a_2P_1 + b_2 = a_2(a_1x + b_1) + b_2$$
$$= a_1a_2x + a_2b_1 + b_2$$

Now, $\delta(P_1P_2) = \overline{P_1} \cdot \overline{P_2} = \overline{P_1P_2}$

Also; $\delta(P_1P_2) = \sigma(P_1)\delta(P_2) + \tau(P_2)\delta(P_1) = \overline{P_1} \cdot \overline{P_2} + 0 \cdot \overline{P_1} = \overline{P_1 P_2}$.

Generally; $\delta(P_1, \dots, P_n) = \overline{P_i}$ where $i = 1,23,\dots,n$ and $\overline{P_i} = a_i x$, also $\sigma(P_i) = \overline{P_i}$ and $\tau(P) = 0$, $P_i, \overline{P_i}, P \in N$, then $\delta$ is the left-sided outer $(\sigma,\tau)$-$n$- derivation on $N$.

The above is the left-sided outer $(\sigma,\tau)$-$n$-derivation on $N$, which could be used to establish commutativity of of special class of right nearrings.

## RESULTS

**Theorem 1:** Assume a prime nearring $N$ and $\delta$ be a positive left-sided outer $(\sigma,\tau)$-$n$-derivation of $N$. If $\delta(N,\dots,N)$ is contained in centre $Z$, it implies $N$ is commutative.

**Theorem 2:** Let $\delta$ be non-zero left-sided outer $(\sigma,\tau)$-n-derivation on the nearring, assume that for any positive integer $i$ with values $1,2,3,\dots,n$. $\delta(x_1, x_2, \dots, [x_i, y_i], \dots, x_n) = 0 \ \forall x_1, x_2, x_3, \dots, x_n \in N$. Implies $(N, \cdot)$ is commutative.

## DISCUSSIONS

### Proofs of Main Results

We establish the following In order to establish our key theorems;

**Lemma 1;** Let $\delta: \underbrace{N \times N \times \dots \times N}_{n-times} \to N$ an n-additive map. It follows that $\delta$ is a left-sided outer $(\sigma,\tau)$-$n$-derivation in $N$ if;

$$\sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) = \tau(y_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) + \sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n)$$

For all $x_i, y_i \in N$, $i$ is $1,2,3,\dots,n$.

**Proof;**

Let $\delta$ be left-sided outer $(\sigma,\tau)$-$n$-derivation, and $\tau(y_i)$ is a distributive element and $x_i, y_i \in N$ where $1 \le i \le n$, then;

$$\delta(x_1, \dots, (x_i + x_i)y_i, \dots, x_n)$$

$$= \sigma(x_i + x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, \dots, (x_i + x_i), \dots, x_n)$$

$$= \sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, \dots, x_i, \dots, x_n) + \tau(y_i)\delta(x_1, \dots, x_i, \dots, x_n) \quad (1)$$

Again,

$$\delta(x_1, \dots, x_iy_i + x_iy_i, \dots, x_n)$$

$$= \delta(x_1, \dots, x_iy_i, \dots, x_n) + \delta(x_1, \dots, x_iy_i, \dots, x_n)$$

$$= \sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, , \dots, x_i, \dots, x_n) + \sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) \quad (2)$$

Comparing (1) and (2) yields;

$$\sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, \dots, x_i, \dots, x_n) = \tau(y_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) + \sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n)$$

Hence proved.

**Lemma 2**: let $\delta$ be left-sided outer $(\sigma,\tau)$-$n$-derivation on $N$. Then

$$u\{\delta(x_1, \dots, x_iy_i, \dots, x_n)\} = u\sigma(x_i)\delta(x_1, \dots, y_i, \dots, x_n) + u\tau(y_i)\delta(x_1, \dots, x_i, \dots, x_n)$$

For all $x_1, x_2, x_3, \dots, x_i, y_i, \dots, x_n \in N \quad 1 \le i \le n$.

**Proof.** Since $\sigma$ is an automorphism upon $N$, $\exists\ v \in N$ as such $\sigma(v) = u \in N$

For all $x_i, y_i \in N \quad 1 \le i \le n$.

$$\delta(x_1, x_2, x_3, \dots, v(x_iy_i), \dots, x_n) = \sigma(v)\delta(x_1, x_2, x_3, \dots, x_iy_i, \dots, x_n) + \tau(x_iy_i)\delta(x_1, x_2, x_3, \dots, v, \dots, x_n)$$

$$= \sigma(v)\delta(x_1, x_2, x_3, \dots, x_iy_i, \dots, x_n) + \tau(x_i)\tau(y_i)\delta(x_1, x_2, x_3, \dots, v, \dots, x_n) \quad (3)$$

Again;

$$\delta(x_1, x_2, x_3, \dots, (vx_i)y_i, \dots, x_n) = \sigma(vx_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n) + \tau(y_i)\delta(x_1, x_2, x_3, \dots, vx_i, \dots, x_n)$$

$$= \sigma(v)\sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n) + \tau(y_i)\{\sigma(v)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) + \tau(x_i)\delta(x_1, x_2, x_3, \dots, v, \dots, x_n)\}$$

$$= \sigma(v)\sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n) + \tau(y_i)\sigma(v)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) + \tau(y_i)\ \tau(x_i)\delta(x_1, x_2, x_3, \dots, v, \dots, x_n)\}$$

$$= \sigma(v)\sigma(x_i)\delta(x_1, x_2, x_3, \dots, y_i, \dots, x_n) + \sigma(v)\tau(y_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) +$$

$$\tau(x_i)\tau(y_i)\,\delta(x_1,x_2,x_3,\dots,v,\dots,x_n)\} \quad (4)$$

Comparing (3) and (4), we have that

$$\sigma(v)\delta(x_1,x_2,x_3,\dots,x_iy_i,\dots,x_n) = \sigma(v)\sigma(x_i)\delta(x_1,x_2,x_3,\dots,y_i,\dots,x_n) + \sigma(v)\tau(y_i)\delta(x_1,\dots,x_i,\dots,x_n)$$

$\Rightarrow$

$$u\delta(x_1,\dots,x_iy_i,\dots,x_n) = u\sigma(x_i)\delta(x_1,\dots,y_i,\dots,x_n) + u\tau(y_i)\delta(x_1,\dots,x_i,\dots,x_n)$$

Hence proved.

**Proof of Theorem 1;**

As $\delta(N,N,N,\dots,N) \subseteq Z$, and $\delta$ is non-zero, then we say that there exist non-zero elements. $x_1,\dots,x_n \in N$, such that; $\delta(x_1,\dots,x_i,\dots,x_n) \subseteq Z\backslash\{0\}$, $i$ ranges from $1,2,\dots,n$.

Then; $\delta(x_1,\dots,x_i,\dots,x_n) + \delta(x_1,\dots,x_i,\dots,x_n) = \delta(x_1,\dots,x_i+x_i,\dots,x_n) \in Z$

By Result# (5), then it means $N$ with + is abelian.

We now need to show the commutativity of $N$.

Let $x,y \in N$. For automorphisms of $N$ are $\sigma$ and $\tau$, $\exists$ $u,v \in N$ such that $\sigma(u) = x$ and $\tau(v) = y$.

Since $\delta(N,\dots,N) \subseteq Z$, then we assume;

$$\delta(x_1,\dots,x_i,\dots,x_n)y = y\delta(x_1,\dots,x_i,\dots,x_n).$$

Take when $i = 1$ changing $x_1$ by $uv$

$$\delta(uv,x_2,x_3,\dots,x_n)y = y\delta(uv,x_2,x_3,\dots,x_n)$$

$$\Rightarrow\{\sigma(u)\delta(v,x_2,\dots,x_n) + \tau(v)\delta(u,x_2,\dots,x_n)\}y = y\{\sigma(u)D(v,x_2,\dots,x_n) + \tau(v)D(u,x_2,\dots,x_n)\}$$

$$\Rightarrow\{x\delta(v,x_2,x_3,\dots,x_n) + y\delta(u,x_2,x_3,\dots,x_n)\}y = y\{x\delta(v,x_2,x_3,\dots,x_n) + y\delta(u,x_2,x_3,\dots,x_n)\}$$

By Lemma 2, we have;

$$x\delta(v,x_2,x_3,\dots,x_n)y + y\delta(u,x_2,x_3,\dots,x_n)y = yx\delta(v,x_2,x_3,\dots,x_n) + yy\delta(u,x_2,x_3,\dots,x_n)$$

$$\Rightarrow x\delta(v,x_2,x_3,\dots,x_n)y = yx\delta(v,x_2,x_3,\dots,x_n)$$

Then we have;

$\delta(v,x_2,x_3,\dots,x_n)(xy-yx) = 0$, for all the elements in $N$

Then $z\delta(v,x_2,x_3,\dots,x_n)(xy - yx) = 0, \forall z \in N$.

By using the hypothesis and $\delta(N,N,\dots,N) \subseteq Z$,

$$\delta(v,x_2,x_3,\dots,x_n)z(xy - yx) = 0, \forall z \in N$$

$$\delta(v,x_2,x_3,\dots,x_n)N(xy - yx) = \{0\}.$$

As $N$ is a prime, it means $\delta(v,x_2,x_3,\dots,x_n) = 0$ or $(xy-yx) = 0$.

If $xy - yx = 0$, then, the nearring is commutative.

As $\delta(v,x_2,x_3,x_4,\dots,x_n) = 0$.

Substitute $v$ with $vx_1$ in the equation above and by the hypothesis, we have;

$$x\delta(vx_1,x_2,x_3,\dots,x_n) = \delta(vx_1,x_2,x_3,\dots,x_n)x.$$

By Lemma 2 and the fact that $\delta(N,N,\dots,N) \subseteq Z$, we have;

$$\delta(x_1,x_2,x_3,\dots,x_n)(xy-yx) = 0.$$

Since $\delta(N,N,\dots,N) \subseteq Z$,

$$\delta(x_1,x_2,x_3,\dots,x_n)N(xy-yx) = \{0\}.$$

Since $\delta(x_1,x_2,x_3,\dots,x_n) \neq 0,\ xy-yx = 0 \Rightarrow xy = yx, \forall x,y \in N$.

$N$ is a commutative nearring as a result.

**Proof of Theorem 2;**

$\forall x_1,x_2,x_3,\dots,x_i,y_i,\dots,x_n \in N, i$ ranges from $1,2,\dots,n$.

$$\delta(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n) = 0$$

$\Rightarrow$

$$\delta(x_1,x_2,\dots,[x_i,x_iy_i],\dots,x_n) = \delta(x_1,x_2,x_3,\dots,x_i[x_i,y_i],\dots,x_n) = \sigma(x_i)\delta(x_1,x_2,x_3,\dots,[x_i,y_i],\dots,x_n) + \tau([x_i,y_i])\delta(x_1,x_2,\dots,x_i,\dots,x_n) = 0$$

By the assumption, we have that

$$\tau([x_i,y_i])\delta(x_1,x_2,x_3,\dots,x_i,\dots,x_n) = 0$$

$\tau(x_iy_i - y_ix_i)\delta(x_1,x_2,\dots,x_i,\dots,x_n) = 0, \quad \forall x_i,y_i \in N\ 1 \leq i \leq n$

As $\tau$ is injective, the above equation becomes;

$$\tau(x_iy_i)\delta(x_1,x_2,x_3,\dots,x_i,\dots,x_n) = \tau(y_ix_i)\delta(x_1,x_2,x_3,\dots,x_i,\dots,x_n)$$

Then for any $u \in N\ \exists v \in N$ such that $\tau(u) = v$

$\tau(x_iu)\delta(x_1,x_2,\dots,x_i,\dots,x_n) = \tau(ux_i)\delta(x_1,x_2,\dots,x_i,\dots,x_n)$

Now;

$0 = \tau(x_i y_i u - u y_i x_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$. $\forall x_i, y_i, u \in N \; 1 \le i \le n$

$= \tau(x_i y_i u - y_i u x_i)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$

$= \tau(x_i y_i u - y_i x_i u)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$. As $\tau$ is an automorphism

$= \tau(x_i y_i - y_i x_i)\tau(u)\delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$, since $\tau(u) = v$

$= \tau(x_i y_i - y_i x_i) v \delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n)$. $\forall x_i, y_i \quad v \in N \; 1 \le i \le n$

$\Rightarrow \; \tau([x_i, y_i]) v \delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) = 0$

$\Rightarrow \tau([x_i, y_i]) N \delta(x_1, x_2, x_3, \dots, x_i, \dots, x_n) = 0$

By primeness of $N$ and the reality that $\delta$ is non-zero left-sided $(\sigma, \tau)$-n-derivation of $N$, then $\tau([x_i, y_i]) = 0$. As $\tau$ is injective, we have that $[x_i, y_i] = 0$, that is $= 0$

$\Rightarrow y_i x_i = x_i y_i \forall x_i y_i \in N$

Hence $N$ is a commutative ring.

## CONCLUSION

In this work, we reviewed some recent papers on $(\sigma, \tau)$-$n$-derivation and outer $(\sigma, \tau)$-$n$-derivation on left-nearrings, which did not discuss right-nearrings. We considered right-nearrings. The concept of left-sided outer $(\sigma, \tau)$-$n$-derivation on right-nearrings is introduced in this study, along with the partial distributive law's proof and the commutativity of nearrings' establishment using the derivation. Further research may be conducted to develop generalized left-sided outer $(\sigma, \tau)$-$n$-derivation on right-nearrings and more.

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