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ORIGINAL RESEARCH ARTICLE

# A Study of Nigeria Monthly Stock Price Index Using ARTFIMA-FIGARCH Hybrid Model

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## ABSTRACT
Long memory is a phenomenon in time series analysis that is exhibited by a slow decay of the autocorrelation function. It has been observed that the presence of long memory in both mean and volatility can complicate model fitting and compromise forecasting reliability. Meanwhile, the Autoregressive Tempered Fractional Integrated Moving Average (ARTFIMA) as a tempered fractionally differenced long memory mean model and the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH), which is a long memory variance model, could not independently and effectively address the challenges of time series data that displayed long memory in mean and volatility. To tackle this challenge, we introduce a hybrid model called the ARTFIMA-FIGARCH by combining ARTFIMA and FIGARCH models using the transformation method under the assumption that the residuals of the ARTFIMA model are non-normal, serially correlated, and heteroscedastic. To evaluate the effectiveness of this model, we employed the Nigerian Monthly Stock Price Index as well as simulated data sets as a testing ground and compared its performance against existing models like ARFIMA, ARTFIMA, and ARFIMA-FIGARCH. The selection of the most suitable model was determined using the Akaike Information Criterion (AIC) and model performance was assessed through various forecast accuracy measures. Our findings demonstrated that ARTFIMA (0,1.06,1)-FIGARCH (1,0.15,1) emerged as the best candidate of the new model and outperformed ARFIMA (1,1.06,0)-FIGARCH (1,0.15,1). Based on the findings of this study, it is concluded that ARTFIMA-FIGARCH is considered to be the most suitable model for studying the mean and volatility of the Nigerian monthly stock price index.

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## INTRODUCTION

The concept of long memory characteristics pertains to the interdependence or connection among data points collected over a time span. In the research by Granger and Joyeux (1980) and subsequently by Hosking (1981), long-term memory characteristics were defined by the gradual decrease in the autocorrelation function's graphical representation within a dataset. This phenomenon led them to suggest the application of fractional differencing in mean models when long memory is identified in time series data. Noteworthy examples of long-memory mean models found in the literature are the Autoregressive Fractional Integrated Moving Average (ARFIMA) model proposed by Granger, Joyeux, and Hosking and the Autoregressive Tempered Fractional Integrated Moving Average (ARTFIMA) model introduced by Meershart *et al*., (2014). Other models in this category include the Semiparametric Fractional Autoregressive (SEMIFAR) model by Beran (1999), the Beta-ARFIMA ($\beta$-ARFIMA) model by Pumi *et al.* (2019), and the ARFURIMA model by Jibrin (2019).

To address long-term memory effects in variations, fractional differencing was incorporated into variance models. An extension of Nelson's (1991) Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) resulted in the creation of the Fractionally Integrated EGARCH (FIEGARCH) model by Bollerslev and Mikkelson in 1996. Another notable model in the realm of long-term memory variance is the fractionally integrated generalized autoregressive conditional heteroscedasticity (FIGARCH) model introduced Ballie *et al.,* in 1996.

Studies have shown that residuals derived from non-stationary mean models with long memory characteristics,

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including ARFIMA and ARTFIMA, as well as other mean models, frequently display serial correlation. This discovery has been documented in research conducted by Zhou and He (2009) and also by Duppati *et al.*, (2016). Furthermore, placing exclusive reliance on long memory variance model such as FIGARCH can result in less accurate predictions. As a result, the integration of mean and variance models into hybrid models, which encompass both mean and variability considerations simultaneously, has the potential to yield enhanced outcomes.

Baillie *et al.*, (1996) made notable contributions to the realm of hybrid modeling by introducing an innovative strategy called the ARFIMA-GARCH model. This hybrid approach was specifically crafted to simultaneously explore patterns of long memory (LM) and variance within the context of inflation trends in the United States. The ARFIMA-GARCH model combines autoregressive fractionally integrated moving average (ARFIMA) models, which effectively capture long memory effects in the mean, with generalized autoregressive conditional heteroscedasticity (GARCH) models, adept at capturing volatility patterns in the variance. Additional studies that have employed hybrid modeling techniques include those by Ishida and Watanabe (2009), Leite *et al.*, (2009), Almeida *et al.*, (2017), Sivakumar and Mohandas (2009), Korkmaz *et al.*, (2009), Ambach and Ambach (2018), Jibrin (2019), and Kabala (2020).

However, the ARTFIMA-FIGARCH hybrid model with tempered fractional differencing for modeling long memory in mean and long memory in volatility can handle time series data whose fractional differencing value *d* can take any value greater than zero, which could not be adequately addressed by previous models. The primary objective is to introduce an innovative and tempered fractionally differenced hybrid model termed ARTFIMA(p,λ,d,q)-FIGARCH(1,1). This hybrid model is specifically designed to effectively tackle the challenge posed by noisy signals that can distort modeling techniques when dealing with both mean and volatile time series exhibiting long-term memory characteristics.

## MATERIALS AND METHODS

The general form of an ARFIMA model of Granger and Joyeux (1980) and Hosking (1981) is given by:

$$\varphi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t,\ \ 0<d<1. \tag{1}$$

The $\varphi(L)$ and $\theta(L)$ are called characteristics polynomial and the $(1-L)^d$ is the fractional operator. The $\varphi_1, \varphi_2, \dots, \varphi_p$ and $\theta_1, \theta_1, \dots, \theta_q$ are unknown parameters and must be estimated from the sample data, *d* is the long memory parameter, $L$ lag operator and $\varepsilon_t$ is the error term.

While the ARTFIMA model of Meerchaert *et al.,* (2014) is defined as follows:

$$\varphi(L)\left(1-e^{-\lambda}L\right)^d Y_t = \theta(L)\varepsilon_t, \tag{2}$$

Where:

$\varphi(L) = 1-\varphi_1 L - \varphi_2 L^2 - \dots - \varphi_p L^p$ and $\theta(L) = 1 + \theta L + \theta_2 L^2 + \dots + \theta_q L^q$ are AR and MA parameters respectively,

$d>0$ is a fractional differencing parameter, and

$\lambda > 0$ is the tampering parameter. It is also called the stability index for measuring the heavy tail of a time series. The $\left(1-e^{-\lambda}L\right)^d$ is a filter for transforming the non-stationary time series $Y_t$.

**Assumptions of the ARTFIMA( p,λ,d,q )-FIGARCH (1,d,1) Model**

a. The current study assumed that the model in (2) failed to completely eliminate the magnitude of trend, heavy tail and long memory known as the component of variation in the time series $Y_1$, $Y_2,\dots, Y_N$. Large proportion of these variations are also found to be present in the residuals $\varepsilon_1$, $\varepsilon_2,\dots, \varepsilon_N$ of the ARTFIMA model in (2).
b. Also, the current study assumed that the residuals from the ARTFIMA model, $\varepsilon_1$, $\varepsilon_2,\dots, \varepsilon_N$ are auto-correlated and heteroscedastic. The Ljung-Box and ARCH-LM test can be used to check the autocorrelation and heteroscedasticity of the residuals. In time series analysis, estimating the ARTFIMA alone would lead to bad modeling and consequently presenting an unreliable forecast.
c. Kabala (2020) considers the residuals of ARTFIMA model and introduced the ARTFIMA-GARCH hybrid model to study both the mean and volatility in time series.

Moreover, Engle (1982) considered $\varepsilon_t$ in eq.(2) to be a stochastic process defined as:

$$\varepsilon_t = a_t\sigma_t. \tag{3}$$

Where $E(a_t)=0$, $Var(a_t)=1$ and $\sigma_t$ is positive and changes with respect to time, t. This implies that the process $\{a_t\}$, is assumed to be serially uncorrelated and expressed as:

$$a_t \sim iid(0,1) \tag{4}$$

FIGARCH(1,d,1) model of Baillie *et al.*, (1996) is defined as:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\alpha(L)(1-L)^d\}\varepsilon_t^2. \tag{5}$$

Where $\alpha$ and $\beta$ are parameters of the model and we assume that all the roots of the polynomials $1-\beta(L)$ and $\alpha(L)$ lie outside the unit circle.

Therefore, ARTFIMA(p, $\lambda$, $d_1$,q)-FIGARCH(1, $d_2$,1) model is represented as

$$Y_t = \frac{\sum_{i=1}^{p}\varphi_i(1-e^{-\lambda}L)^{d_1}Y_{t-i}+\sum_{i=1}^{q}\theta_j\varepsilon_{t-j}+\sqrt{(\omega^*+\alpha^*)}a_t}{(1-e^{-\lambda}L)^{d_1}} \quad (6)$$

Note $\omega^* = \omega(1-\beta(L))^{-1}$, $\alpha^* = [1-[1-\beta(L)]^{-1}\alpha(L)(1-L)^{d_2}]\varepsilon_t^2$

where

$\varphi, \theta, \alpha, \beta, \lambda, d_1$ and $d_2$ are parameters of the model to be estimated.

## PROPERTIES OF THE MODEL

This subsection deals with properties of the ARTFIMA-FIGARCH Model.

### Mean

Consider ARTFIMA $(1, \lambda, d_1, 1)$-FIGARCH$(1, d_2, 1)$

$$Y_t(1-e^{-\lambda}L)^d = \varphi_1(1-e^{-\lambda}L)^d Y_{t-1} + \theta_1\varepsilon_{t-1} + \sqrt{(\omega^*+\alpha^*)}a_t \quad (7)$$

$$\mu = 0 \quad (8)$$

**Note**: $E[Y_t] = E[Y_{t-1}] = \mu$ and $E[\varepsilon_{t-1}] = E[a_t] = 0$

∴ ARTFIMA- FIGARCH is a zero mean Process.

### Variance

To obtain the variance of the model, we first multiply equation (7) by $Y_t$ and take its expectation:

$$E[Y_t^2] = \varphi_1^2 E[Y_{t-1}^2] + 2\varphi_1\theta_1(1-e^{-\lambda}L)^{-d}E[\varepsilon_{t-1}Y_{t-1}] + 2\varphi_1(1-e^{-\lambda}L)^{-d}\sqrt{(\omega^*+\alpha^*)}E[a_tY_{t-1}] + 2\theta_1((1-e^{-\lambda}L)^{-d}))^2\sqrt{(\omega^*+\alpha^*)}E[a_t\varepsilon_{t-1}] + \theta_1^2((1-e^{-\lambda}L)^{-d}))^2E[\varepsilon_{t-1}^2] + ((1-e^{-\lambda}L)^{-d}))^2(\omega^*+\alpha^*)E[a_t^2] \quad (9)$$

**Note:**

$$E[Y_t^2] = E[Y_{t-1}^2] = \gamma_0 \text{ and } E[\varepsilon_{t-1}Y_{t-1}] = E[a_tY_t] = \sigma_\varepsilon^2$$

$$\gamma_0 = \frac{\sigma_\varepsilon^2[2\varphi_1\theta_1+\theta_1^2(1-e^{-\lambda}L)^{-d}+(\omega^*+\alpha^*)(1-e^{-\lambda}L)^{-d}]}{(1-e^{-\lambda}L)^d(1-\varphi_1^2)} \quad (10)$$

The variance of ARTFIMA-FIGARCH Process is given in equation (10) above.

### Autocovariance at Lag 1

To obtain Autocovariance at lag 1, multiply equation (7) $Y_{t-1}$ and taking its expectation:

$$\gamma_1 = \frac{\sigma_\varepsilon^2\left(\varphi_1\sqrt{(\omega^*+\alpha^*)}+\theta_1(1-e^{-\lambda})^{-d}\right)}{(1-e^{-\lambda}L)^d(1-\varphi_1^2)} \quad (11)$$

### Autocorrelation at Lag1 ( $\rho_1$ )

Finally the autocorrelation at lag 1 ( $\rho_1$ ) of the new model is given as

$$\rho_1 = \frac{\varphi_1\sqrt{(\omega^*+\alpha^*)}+\theta_1(1-e^{-\lambda}L)^{-d}}{2\varphi_1\theta_1+\theta_1^2(1-e^{-\lambda}L)^{-d}+(\omega^*+\alpha^*)(1-e^{-\lambda}L)^{-d}} \quad (12)$$

## SIMULATION STUDY

In this part of the study, we employed Monte Carlo simulation to create datasets of various sizes (100, 200, 500 and 1000) for ARTFIMA modeling. We duplicated these datasets for use in the ARTFIMA-FIGARCH and ARFIMA-FIGARCH estimations. For the hybrid models, we conducted the estimation procedure iteratively, investigating a range of p and q combinations. These values of p and q were both limited to 3 or lower, thereby keeping their sum under 5 during the hybrid model estimation process.

Compared to a significance level of 0.05, the results from conducting portmanteau and ARCH-LM tests on the four simulated datasets, as displayed in Table 1, indicate that the residuals of the ARTFIMA model exhibit heteroscedasticity and serial correlation. This suggests that relying solely on the ARTFIMA model is insufficient for analyzing the dataset, primarily due to the presence of noise signals. Consequently, it is advisable to incorporate a variance model, and in this regard, FIGARCH is being considered as a suitable option.

In summary, the outcomes presented in Table 2 demonstrated that the ARTFIMA- FIGARCH model surpasses the existing ARFIMA- FIGARCH model in terms of its ability to accurately represent the data's fit and its forecasting precision. Consequently, integrating the FIGARCH variance model into the ARTFIMA framework has brought about notable enhancements. As a result, the hybrid model becomes a more appropriate option for both the analysis of the generated data and the generation of more precise predictions.

Table 1 Simulation result of ARTFIMA Model for sample sizes n =100, 200, 500 and 1000

| ARTFIMA (p,d,q) | Loglikelihood | AIC | Portmanteau test | ARCH-LM Test |
|---|---|---|---|---|
| **n=100** | | | | |
| ARTFIMA (1,d,0) | -718.1677 | 1438.335 | 0.03983 | 0.03589 |
| ARTFIMA (0,d,1) | -704.4607 | 1410.921 | 0.04688 | 0.02891 |
| ARTFIMA (1,d,1) | -704.3798 | 1412.76 | 0.01001 | 0.03851 |
| ARTFIMA (2,d,0) | -706.8703 | 1417.74 | 0.03083 | 0.03475 |
| ARTFIMA (0,d,2) | -706.4651 | 1416.93 | 0.03928 | 0.06922 |
| **n=200** | | | | |
| ARTFIMA (1,d,0) | -1406.401 | 735.2701 | 0.03665 | 0.02867 |
| ARTFIMA (0,d,1) | -1406.254 | 735.3077 | 0.02885 | 0.06333 |
| ARTFIMA (1,d,1) | -1406.538 | 737.2214 | 0.04162 | 0.01277 |
| ARTFIMA (2,d,0) | -1406.287 | 738.0678 | 0.03576 | 0.02776 |
| ARTFIMA (0,d,2) | -1405.923 | 737.784 | 0.0005376 | 0.02623 |
| **n=500** | | | | |
| ARTFIMA (1,d,0) | -3526.924 | 7284.098 | 5.251e-10 | 0.015 |
| ARTFIMA (0,d,1) | -3526.915 | 7085.71 | 3.62e-06 | 0.01553 |
| ARTFIMA (1,d,1) | -3525.317 | 7087.665 | 3.36e-06 | 0.01456 |
| ARTFIMA (2,d,0) | -3526.8 | 7237.371 | 2.419e-05 | 0.04375 |
| ARTFIMA (0,d,2) | -3526.71 | 7090.789 | 7.811e-06 | 0.02 |
| **n=1000** | | | | |
| ARTFIMA (1,d,0) | -7280.815 | 14563.63 | 0.001059 | 2.2e-16 |
| ARTFIMA (0,d,1) | -7093.783 | 14189.57 | 1.14e-12 | 0.01112 |
| ARTFIMA (1,d,1) | -7093.544 | 14191.09 | 1.05e-12 | 0.0111 |
| ARTFIMA (2,d,0) | -7214.265 | 14432.53 | 4.378e-12 | 0.05103 |
| ARTFIMA (0,d,2) | -7103.693 | 14211.39 | 3.129e-12 | 0.01123 |

Table 2: Simulation result of ARFIMA- FIGARCH and ARTFIMA- FIGARCH models for sample sizes n =100, 200, 500 and 1000

| MODEL | AIC | MAE | MSE | RMSE |
|---|---|---|---|---|
| **n=100** | | | | |
| **ARTFIMA(0,d,1)-FIGARCH(1,d,1)** | **13.905** | **0.8913** | **1.1328** | **1.0643** |
| ARFIMA(2,d,3)-FIGARCH(1,d,1) | 13.893 | 249.3704 | 82376.22 | 287.0126 |
| **n=200** | | | | |
| **ARTFIMA(0,d,1)-FIGARCH(1,d,1)** | **13.865** | **0.9034** | **1.1277** | **1.0619** |
| ARFIMA(1,d,0)-FIGARCH(1,d,1) | 13.872 | 249.7105 | 81900.88 | 286.1833 |
| **n=500** | | | | |
| **ARTFIMA(2,d,1)-FIGARCH(1,d,1)** | **13.859** | **0.8848** | **1.0318** | **1.0158** |
| ARFIMA(2,d,2)-FIGARCH(1,d,1) | 13.877 | 251.7693 | 83678.29 | 289.272 |
| **n=1000** | | | | |
| **ARTFIMA(2,d,2)-FIGARCH(1,d,1)** | **13.842** | **0.8741** | **1.0176** | **1.0088** |
| ARFIMA(1,d,0)-FIGARCH(1,d,1) | 13.852 | 250.0027 | 83333.03 | 288.6746 |

## APPLICATION

This section presents the application of the ARTFIMA, ARFIMA, ARTFIMA-FIGARCH and ARFIMA-FIGARCH model by using Nigerian Monthly Stock Price Index obtained from Morgan Stanley Capital Index (MSCI) from 2012 to 2020.

Illustrated in Figure 1 are the time series plots of Nigeria Monthly Stock Price Index. These plots reveal fluctuations, deterministic trends, and nonlinearity within the data. The observed trend behavior stems from abrupt shifts in the series. Furthermore, the Autocorrelation Function (ACF) demonstrates a gradual decay, which signifies the presence of a long-range dependence process. As a result, there are indications of Long Memory (LM) characteristics in the studied series. Such series exhibiting LM traits often yield LM values within the range of $0 < d < 1$ and beyond. Hence, the series can be characterized as originating from a long memory process. On average, the series lacks stationarity.

Figure 2 illustrates the plot of log-returns and the Autocorrelation Function (ACF) for the Nigerian Monthly Stock Price Index. The log-returns demonstrate intervals of heightened volatility, providing evidence of volatility clustering. This phenomenon implies that returns tend to group together over time, with periods of increased volatility followed by comparatively less volatile periods. Moreover, the ACF exhibited in Figure 2 for the Nigerian Monthly Stock Price Index showcases consistent and positive values that are notably elevated. The gradual decline of the ACF towards zero signifies the presence of volatility clustering. This indicates that the autocorrelation of the returns remains significant across numerous time lags, underscoring that past returns maintain a correlation with future returns. This pattern is a fundamental characteristic of volatility clustering within financial time series data.

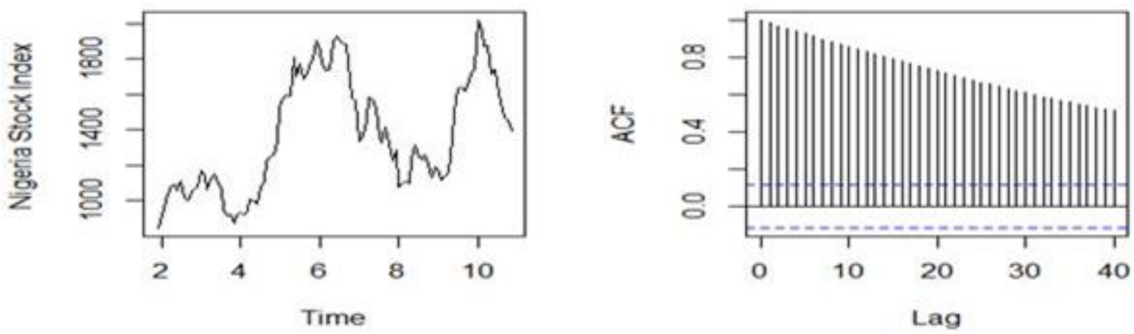


**Figure 1**: Time Series Plot and ACF for Nigeria Monthly Stock Price Index.

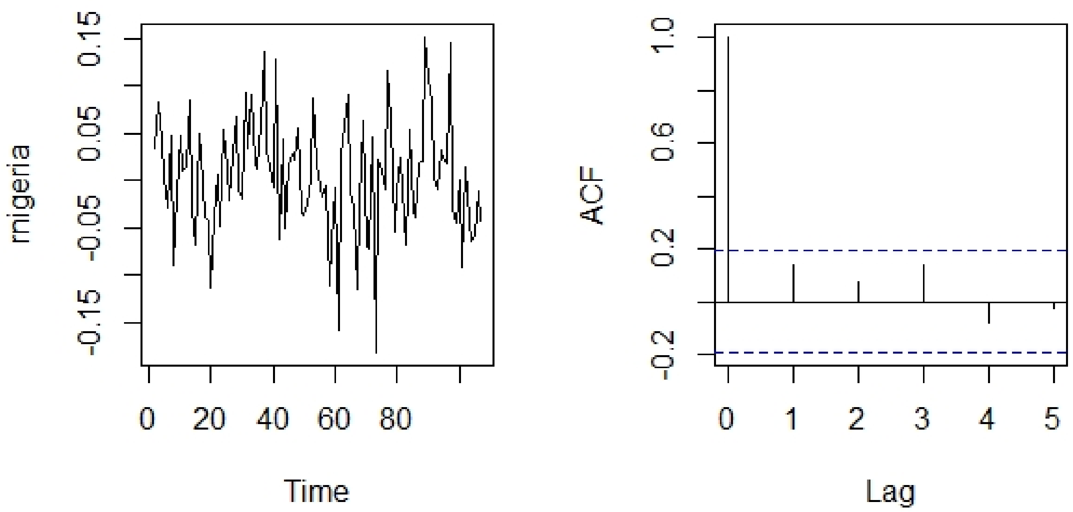


Figure 2: The plot of log-returns and ACF for Nigerian Monthly Stock Price Index.

## MEAN MODELING

In this section, we identify the parameters for each set of candidate from the mean models ARFIMA and ARTFIMA models putting into consideration the model from the estimation with least AIC values. Table 3 displays the outcomes of the analysis carried out on Nigerian Monthly Stock Price Index dataset using the two mean models.

Table 3: AIC Values and Diagnostic tests for ARFIMA and ARTFIMA Models

| ARFIMA | | | | ARTFIMA | | | |
|---|---|---|---|---|---|---|---|
| ARFIMA (p,d,q) | AIC | Portmanteau Test | ARCH-LM Test | ARTFIMA (p,$\lambda$,d,q) | AIC | Portmanteau Test | ARCH-LM Test |
| ARFIMA (1,d,0) | **1257.452** | 2.2e-16 | 2.2e-16 | ARTFIMA (1,d,0) | 1222.666 | 0.0004071 | 0.002454 |
| ARFIMA (0,d,1) | 1299.126 | 2.2e-16 | 2.2e-16 | ARTFIMA (0,d,1) | **1222.662** | 0.0003534 | 0.04523 |
| ARFIMA (1,d,1) | 1259.139 | 2.2e-16 | 2.2e-16 | ARTFIMA (1,d,1) | 1224.117 | 0.0004702 | 0.002485 |
| ARFIMA (2,d,0) | 1259.814 | 2.2e-16 | 2.2e-16 | ARTFIMA (2,d,0) | 1224.541 | 0.0007355 | 0.01777 |
| ARFIMA (0,d,2) | 1288.923 | 2.2e-16 | 2.2e-16 | ARTFIMA (0,d,2) | 1224.313 | 0.001543 | 0.003287 |

The AIC values were compared and candidate models with the minimum AIC values are identified. Results indicated that the ARTFIMA have the least AIC values. Therefore, the ARTFIMA models are a better fit for the data when compared with ARFIMA models. However, results showed evidence of serial correlation and heteroscedasticity in residuals of both ARFIMA and ARTFIMA models because the p-values are less than 0.05. The residuals of the two mean models are serially correlated indicating that they are not appropriate for the data set. In view of these, the fractionally integrated volatility model, FIGARCH is considered next to form hybrid models with the ARFIMA and ARTFIMA models. The reason for introducing the variance model is to improve the model fitting to the data. Therefore, further analyses based on ARFIMA-FIGARCH and ARTFIMA-FIGARCH models are carried out and discussed in next section.

## HYBRID MODELING

The analysis for ARFIMA-FIGARCH and ARTFIMA-FIGARCH models for the Nigerian Monthly Stock Price Index is shown in Table 3 below. The two models are used for the estimation, and the procedures involved shall be repeated for $p \leq 3$ and $q \leq 3$ so that $p+q \leq 5$ in the hybrid models, estimations and the residuals are assumed to be distributed normal (norm), student-t (std), skewed student-t (sstd) and generalized error distribution (ged). AIC values, log likelihood values and measures of forecast accuracy; Mean Absolute Error (MAE), Mean Square Error (MSE) and Root Mean Square Error (RMSE) shall considered.

### The Hybrid Models Identification

After going through the procedures of the model diagnostic test, the best models for the two hybrid models are identified for Nigerian Monthly Stock Price Index. The identification was done and selection was based on the models that are stationary and have the least AIC values. The outcomes are as presented in Table 4.

Table 4: Estimation of ARFIMA(*p,d,q*) –FIGARCH(1,1)and ARTFIMA (p,λ,d,q)-FIGARCH(1,1) with their AIC values.

| Model | Parameters | Estimate | AIC | MAE | MSE | RMSE |
|---|---|---|---|---|---|---|
| ARTFIMA(0,1.06,1)-FIGARCH(1,0.15,1) | MA1 | 0.0426 | 11.465 | 0.6902 | 0.8420 | 0.0426 |
| | $\lambda$ | 1.2 | | | | |
| | $\alpha$ | 0.0101 | | | | |
| | $\beta$ | 0.0398 | | | | |
| ARFIMA(1,1.06,0)-FIGARCH(1,0.15,1) | AR1 | 0.9878 | 11.788 | 63.7709 | 7231.032 | 85.0355 |
| | $\alpha$ | 0.0002 | | | | |
| | $\beta$ | 0.0400 | | | | |

The ARTFIMA-FIGARCH model exhibits the smallest AIC value as well as having lower forecast accuracy measures (MAE, MSE, and RMSE) values in comparison with the corresponding values of the ARFIMA-FIGARCH model. In particular, the best candidate of ARTFIMA-FIGARCH, ARTFIMA(0,1.06,1)-FIGARCH(1,0.15,1) model has the lowest AIC value (11.465) in addition to recording the smallest MAE, MSE, and RMSE values (0.6902, 0.8420 and 0.0426) as compared to ARFIMA(1,1.06,0)-FIGARCH (1,0.15,1) with AIC value (11.788) and MAE, MSE, and RMSE values (63.7709, 7231.032, 85.0355). This indicates that the ARTFIMA-FIGARCH model is more suitable for fitting the dataset and making a more reliable forecast as compared to the ARFIMA-FIGARCH model of Ballie *et al.,* (1996).

## CONCLUSION

This study introduces a novel hybrid model named ARTFIMA-FIGARCH and carried out a comprehensive comparative analysis against established mean and hybrid models. The outcomes show the superior suitability and performance of the hybrid model, specifically ARTFIMA-FIGARCH, in comparison with the mean models (ARTFIMA and ARFIMA) and the hybrid model, ARFIMA-FIGARCH, for both simulated and real-world datasets. This study is in conformity with the works of Jibrin (2019) and Kabala (2020), who are of the opinion that hybrid models outperform mean models. These findings clearly show the dominance of the ARTFIMA-FIGARCH model as a hybrid mean-volatility model over its counterpart, the ARFIMA-FIGARCH model, and are therefore considered the most suitable for studying the mean and volatility of the Nigerian Monthly Stock Price Index and other financial data that exhibit similar characteristics.

## REFERENCES


Almeida, R., Dias, C., Silva, M. E. & Rocha, A.P. (2017). ARFIMA-GARCH Modeling of HRV: Clinical Application in Acute Brain Injury. Complexity and nonlinearity in cardio-vascular. *Signal-Springer,* 451-468. **[Crossref]**

Ambach, D., & Ambach, O. (2018, August). Forecasting the oil price with a periodic regression ARFIMA-GARCH process. In *2018 IEEE Second International Conference on Data Stream Mining & Processing (DSMP)* (pp.). IEEE. **[Crossref]**

Ambach, D., & Ambach, O. (2018). Forcasting the oil price with a periodic Regression ARFIMA-GARCH Process. *IEEE Second International Conference on data stream minning and Processing (DSMP),* 212-217. **[Crossref]**

Ballie, R.T, Chung, C.F, & Tieslau, M.A.(1996a). Analysing Inflation by the Fractionally Integrated ARFIMA-GARCH Model. *Journal of Applied Econometrics* 11(**1**), 23-40. **[Crossref]**

Ballie, R.T., Bollesleve, T., & Mikkelsen, H.O. (1996b). Fractionally integrated generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 74(**1**), 3-30. **[Crossref]**

Beran, J. (1999). SEMIFAR models a semiparametric fractional framework for modelling trends, long-range dependence and nonstationarity. *Preprint, University of Konstanz.* *hdl.handle.net*

Bollerslev, T., & Mikkelsen, H. O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of econometrics,* 73**(1),** 151-184. **[Crossref]**

Cont, R., & Tankov, P., (2004), Financial modeling with Jump Processes. *Journal of Applied Mathematics and Physics* 2**(11)** **[Crossref]**

Ding, Z., Granger, C.W. J., & Engle, R.F. (1993) Along memory property of stock market returns and a new model. *Journal of empirical finance*, 1(**1**), 83-106. **[Crossref]**

Duppati, G., Kumar, A. S., Scrimgeour, F., & Li, L. (2017). Long memory volatility in Asian Stock Market. *Pacific Accounting review* 29**(3)**, 423- 442. **[Crossref]**

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of Variance of UK Inflation. *Econometrica*, 50, 987-1008. **[Crossref]**

Granger, C.W.J., & Joyeux R., (1980). An Introduction to Long Memory Time Series Models and Fractional Differencing. *Journal of Time Series Analysis* 1 (1)15-29. **[Crossref]**

Hosking J.R.M. (1981). Fractional Differencing. Biometrika 68**(1)**, 165-176. **[Crossref]**

Ishida, I., & Watanabe, T. (2009). Modeling and forecasting the volatility of the Nikkei 225: Realized volatility using ARFIMA-GARCH. *Research unit for statistical and empirical Analysis for social sciences (HI-Sat) discussion paper series* 032. ideas.repec.org

Jibrin, S. A., (2019). Interminable long memory model and its hybrid for time series modeling, School of Mathematical Sciences, Universiti Sains Malaysia, Pulau- Penang, Malaysia, Unpublished Ph.d thesis.

Kabala, J., (2020). ARTFIMA processes and their applications to solar flare data. *Creative components*. 595. dr.lib.iastate.edu

Korkmaz, T., Cevik, E.I., & Ozatac, N. (2009). Studying long Memory in ISE using ARFIMA-FIGARCH Model and Structural Break Test. *International Research Journal of Finance and Economics,* 26, 186-191. ideas.repec.org

Leita, A., Rocha, A., & Silva, M. (2009). Long Memory and Volatility in HRV: An ARFFIMA-GARCH Approach. *Computers in Cardiolgy* , 165- 168. **ISSN:** 2325-8853

Meerschaert, M.M., Sabzikar ,F., Panikumar, M.S., & Zeleke, A., (2014). Tempered Fractional Time Series Model for turbulence in Geophysical flows. *Journal of Statistical Mechanics: Theory and Experiment,* 9. **[Crossref]**

Nelson, D. B., (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59**(2)**, 347-370. **[Crossref]**

Pumi, G., Valk, M., Bisognin, C., Bayer, F. M., & Prass, T. S. (2019). Beta autoregressive fractionally integrated moving average models. *Journal of Statistical Planning and Inference*, 200, 196-212. **[Crossref]**

Rahman R.A., & Jibril S.A. (2018). A modified long memory model for modeling interminable Long memory process. *International conference on computing, mathematics and Statistics.* 235-243. **[Crossref]**

Safadi, T., & Pereira, I. (2010). Bayesian analysis of FIAPARCH model: an application to Sao Paulo stock market. *Int. J. Stat. Econ*, *5(**10**)*, 49-63.

Sivakumar, P. B., & Mohandas, V. P. (2009). Modeling and Predicting Stock Returns Using ARFIMA-FIGARCH: A case study on Indian stock data. Conference paper: *World Congress On Nature and Biologically inspired Computing*,Coimbatore, India. **[Crossref]**

Slaveya, Z., (2018). ARFIMA-FIGARCH, HYGARCH and FIAPARCH Models of Exchange Rates, *Journal of the Union of Scientists,* 7**(2)**, 142-1 53. ideas.repec.org

Tse, Y.K., (1998). The Conditional Heteroscedasticity of the Yen- Dollar Exchange Rate. The *Journal of Applied Econometrics*, 13**(1)**, 49-55. **[Crossref]**

Zhou, J., & He, C., (2009). Modeling S& P 500 STOCK INDEX using ARMA-ASYMMETRIC POWER ARCH models. Unpublished Master thesis.