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ORIGINAL RESEARCH ARTICLE

Extreme Rainfall Modeling through the Lens of Extreme Value Theory: A Case Study of Katsina City, Nigeria.

Ibrahim Lawal Kane[1]*, Khadija Tahir Umar[2], Anasu Rabe[3].
[1,2,3]Department of Mathematics and Statistics, Umaru Musa Yar'adua University, Katsina, 2218, Nigeria.

ABSTRACT

Climate change has conveyed about strange new weather patterns, among others is changes in rainfall extremes. To make adequate inferences about extreme rainfall, it's important to invest in meteorological research. This can include the use of climate models to predict extreme rainfall events with greater accuracy. Additionally, information sharing can help ensure that countries are well-prepared for extreme weather events and can collaborate to address the global challenges posed by climate change. Monthly rainfall record of Katsina city was collected from the Nigerian Meteorological Agency (NiMet), Nigeria. In pursuit of this objective, the study employed Extreme Value Theory to model and forecast extreme rainfall of Katsina city, utilizing rainfall data spanning from 1989 to 2019. The research employed the Maximum Likelihood Estimation method to derive the model parameters. For the Generalized Extreme Value Distribution (GEVD), the block maxima approach was applied, whereas the Generalized Pareto Distribution (GPD) was fitted using the Peak Over Threshold method. The analysis revealed that the optimal model within the GEVD framework was the Frechet distribution, whereas the ordinary Pareto distribution emerged as the optimal model when considering values above the threshold. Additionally, the study included predictions for return periods of 2, 20, and 100 years based on the return level estimates, accompanied by the presentation of their respective confidence intervals. The analysis demonstrated that as the return periods lengthened, there was a proportional increase in the return levels. The study's model diagnostics, involving probability, density, quantile, and return level plots, collectively indicated that the provided models were well-suited for the dataset..

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KEYWORDS

Extreme Value Theory, Generalized Extreme Value Distribution, Extreme Rainfall, Peak Over Threshold. Generalized Pareto Distribution.

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INTRODUCTION

The adjective "extremes" encompass a broad range of rare phenomena that, due to their exceptional nature, can result in catastrophic economic and human consequences. Extreme weather occurrences, including heavy precipitation events, exert adverse effects on our society by impacting agriculture and the economy (Rosenzweig, 2001). For example, in Nigeria, the 2017 flooding was raised to as the most destructive in the last 40years, and was ascribed to a mixture of two events; local heavy rainfall and the discharge of excess water from the Lagdo dam in the neighboring Cameroon Republic, the flood have ravaged 27 states which include among others; Enugu, Cross River, Lagos, Kwara, Zamfara, Adamawa Imo, Kogi, Jigawa, Taraba, Ondo, Rivers, Katsina, Bauchi, Gombe, Akwa ibom, Anambra, Ogun, Niger, Delta, Yobe, Ebonyi, Enugu, Nasarawa and Osun in which no fewer than 90 people died (NEMA, 2017).

In 2022, a total of 24 individuals tragically lost their lives due to the devastating floods and windstorms that swept through all 34 local government areas of Katsina State. According to the State Emergency Management Agency, this incident had a profound impact, affecting more than 18,245 individuals and causing damage to 16,625 houses. Additionally, 1,620 farmlands were submerged in Kafur, Danja, and Ingawa LGAs (Punch Newspaper, October 8, 2022). Hence, knowing how often such extreme weather condition occurs is of practical significance for planning and mitigation purposes. Extreme weather results to events such as floods, droughts, salinization/contamination of agricultural water likely results to decrease in agricultural productivity in already fragile areas, especially in Sub-Saharan African Semi-Arid and Arid regions. Extreme Value Theory (EVT) is extensively applied in hydrology and climatology. Results obtained from EVT in engineering practice play an essential role in flood management and water resources

Correspondence: Ibrahim Lawal Kane. Department of Mathematics and Statistics, Umaru Musa Yar'adua University, Katsina, 2218, Nigeria. ✉ Ibrahim.lawal@umyu.edu.ng. Phone Number: +234 703 663 6088.
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design (Katz & Philippe, 2002). Several researchers have provided useful applications of extreme value distributions particularly to climate data from different regions of the world, for example (Wang et al., 2015; Kilavi et al., 2018; Faithful et al., 2019; Carlos et al,. 2022).

The essential evolution of extreme precipitation is in itself a subject of interest. Apart from its climatological significance, it can find practical use, for instance as input for flood models among other parameters. The purpose of explicitly modeling precipitation data is to shed light on the seasonal behavior of extreme rainfall.

This paper aimed to develop a model that will capture the extreme dynamics of rainfall in Katsina specifically, to discover the suitable distributions of the rainfall and to predict the extreme rainfall return periods with their corresponding return levels. Thus, to the best of our knowledge little or not has been done on modeling extreme rainfall in Katsina paper using EVT. The results of this paper will significantly help researchers in engineering and climatology, risk management and decision-makers with awareness about the behavior of rainfall and to aid them derive up appropriate plans, policies to prepare for challenges due to extreme rainfall.

MATERIALS AND METHODS

The purposes of many researches may not be attained without the analysis of empirical data. In this paper, monthly rainfall amount records spanned from 1989 to 2019 obtained from the Nigerian meteorological Agency was used. Extreme value analysis using the Generalized Pareto Distribution (GPD) and Generalized Extreme Value distribution (GEV) were fitted using method of maximum likelihood estimates (MLE).

The Study Area

Katsina is a state located in northern Nigeria. It occupies 24,192 square kilometres and bounded in the East by Jigawa and Kano States, in the West by Zamfara State, in the South by Kaduna State and in the North by the Niger Republic. The indigenes of Katsina are Hausa and Fulani. The city is the center of an agricultural region production of groundnuts, cotton, hides, millet and guinea corn and also has mills for producing peanut oil, and steel, it is also a center for large scale poultry, farming and rearing cows, goats and sheep.

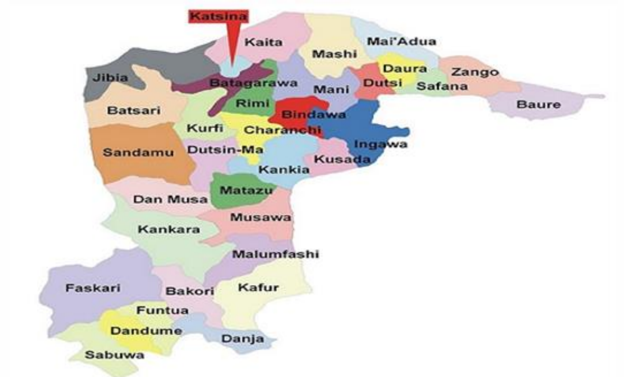

Figure 1: Map of Katsina (http/:katsina map)

Generalized Extreme Value Distribution

Suppose that $X_1, X_2, \ldots$ are independently identically distributed (iid) random variables with common cumulative distribution function $(cdf)\ F.\ Let\ M_n = \max\{X_1, \ldots\ldots, X_n\}$ denote the maximum of the first n random variables and $Let\ w(F) = \sup\{x: F(x) < 1\}$ denote the upper end point of F. Since

$$\Pr(M_n \le x) = \Pr(X_1 \le x \ldots\ldots. X_n \le x) = F^n(x)$$

The X_i commonly signifies value of a process measured over time scale, that is, the rainfall amount recorded within a month period and the M_n represents the monthly maximum rainfall recorded.

The class of extreme value distributions EVDs involved three types of distributions as type I, II and III, where;

i. The type I is the Gumbel Distribution

ii. The type II is the Frechet Distribution

iii. The type III is the Weibull Distribution.

The three distributions are mutually obtainable as family of model referred to as the generalized extreme value family with distribution functions of the form;

$$G(x) = \exp\{-(1 + \epsilon\left(\frac{x-u}{\alpha}\right)\}^{-1/\varepsilon}$$

The family distribution is also recognized as the generalized extreme value distribution denoted by GEV (μ, σ, ξ). The parameters μ, ξ and σ are location, shape and scale parameters respectively. The location parameter determines the most likely value (center) of the distribution. The shape parameter determines the width of the distribution while the scale parameter determines whether the distribution has an upper, lower or end point.

σ parameter distinguishes the three sub distributions from one another, when

$\sigma > 0$ is Frechet Distribution

$\sigma < 0$ 0 Weibull Distribution and

$\sigma = 0$ is Gumbell Distribution

The family of distribution defined by above is known as Generalized Pareto Family, and the distribution itself is often referred to as the Generalized Pareto Distribution.

Generalized Pareto Distribution

Balkema and de Haan (1974) and Pickands (1975) show that F ∈ D (G), where G is GEV(μ, σ ξ) for some $\mu, \sigma\ and\ \varepsilon$ σ and ε and only if

$$\lim_{t \to w(F)} \sup_{0<x<w(F)-t} \left| \frac{1-F(1+x)}{1-F(t)} - \left\{1 + \epsilon \frac{\varphi}{a(t)}\right\}^{-1/t} \right|$$

The second limiting term within the suprema is the survivor function of the Generalized Pareto (GP) distribution and is defined when either $0 < x < \infty$ ($\varepsilon \geq 0$) or $0 < x < -\sigma * (t) / \varepsilon$ ($\varepsilon < 0$),. The case $\varepsilon = 0$ again interpreted as the limit $\varepsilon \to 0$ and thus we have the exponential distribution with mean $\sigma * (t)$ as a special case. We again refer to ε and σ as the shape and scale parameters, respectively.

For a random sample $X_1 \ldots \ldots X_2, \ldots \ldots \ldots, X_n$ with common df and $F\left\{XL_{n,}n \geq 0\right\}$ denote the sequence of record values, where $L_o = 1$ and $L_n = \min((i|i > L_n - 1, Xi > XL_n - 1), n \geq 1$. Then Nagaraja (1977) shows that if for some constants p and q

$E\left(\frac{X_{L1}}{X_{L0}} = x\right) = px + q$ then except for change location and scale.

The GPD includes three types of distributions.

$\sigma = 0$, Exponential distribution.

$\sigma > 0$, Ordinary Pareto distribution.

$\sigma < 0$, Pareto-II type distribution

Techniques for selecting extremes

Extreme value analysis was performed on this study by fitting both the generalized extreme value distribution and Generalized Pareto Distribution using method of maximum likelihood estimates (MLE). In order to model extreme events, it is important that one selects extreme values from the data. There are two methods for identifying extremes in a dataset, namely, Peak over threshold method and Block Maxima method.

Peak over threshold (POT)

The POT model was introduced by Goda (1988), and has been developed in this study for further evaluation. The model can be developed with a long length dataset to obtain the results in a smaller range of confidence intervals (Goda, 2010). The method considers the distribution of exceedances over a certain threshold. The interest in the use of the POT method is estimating the distribution function of values of x above a certain threshold U.

The distribution function Fu is called the conditional excess distribution function and is defined as

$$F_\sigma(x) = P(Y - u \leq x ! Y > u)\ 0 \leq x \leq y, -u$$

where Y is a random variable, it represent the probability that the value of Y exceeds the threshold u by the most amount,x=Y-u. However, we can verify that Fu can be written in terms of F, that is

$$F_a(x) = \left(\frac{F(u+x)-F(u)}{1-F(u)} = \frac{F(y)-F(u)}{1-F(u)}\right)$$

The issue of threshold selection is similar of selection of block size in the block maxima approach. The choice of threshold is not straightforward and usually a compromise has to be found. A high threshold value reduces the bias as this satisfies the convergence towards the extreme value theory but however increases the variance for the estimators of the parameters of the GPD, as there will be fewer data from which to estimate the parameters. A low threshold value on the other hand, results in the opposite i.e., more data with which to estimate the parameters. Consequently, various graphical techniques have been proposed for use in selecting an appropriate threshold. This includes mean excess plot, parameter stability plot and selection based on empirical quantities.

Block maxima approach

The block maxima approach consists of dividing the observation period into non overlapping blocks of equal size and restricts attention to the maximum observation in each block. Larger blocks mean fewer maxima and larger variance in estimation, while smaller blocks may not fit the GEV distribution and lead to bias. Often block size is determined from the pragmatic perspective or data availability, leading to data such as annual, monthly, or daily maxima.

Return level Estimate

Return level is the level that is expected to be exceeded on an average of once every time periods with probability of p. in this study, the return level is the maximum temperature amount and t corresponds to the selection intervals.

$$G_{\varepsilon\mu\sigma} \times exp\left(-\left|\{1 + \varepsilon(\frac{x-u}{\sigma}\right|\})^{-\frac{1}{\varepsilon}}\right) = -p$$

RESULTS AND DISCUSSION

This section presents the analysis of the data and it begins with basic statistical analysis involving graphical tools and basic statistics. In addition, the extreme value analysis of the rainfall is presented. R software was employed in the data analysis.

Rainfall Data

Figure 4 display the rainfall data, based on the plot the data fluctuate over time with some extremes occurring over time in Katsina. The highest maximum rainfall occurred in September 2018 with a recorded magnitude of 379.8mm. Similarly, the second highest amount rainfall occurred in August 2007 with a recorded magnitude of 353.4mm. These extremes rainfall brought about flooding leading to damages to infrastructures.

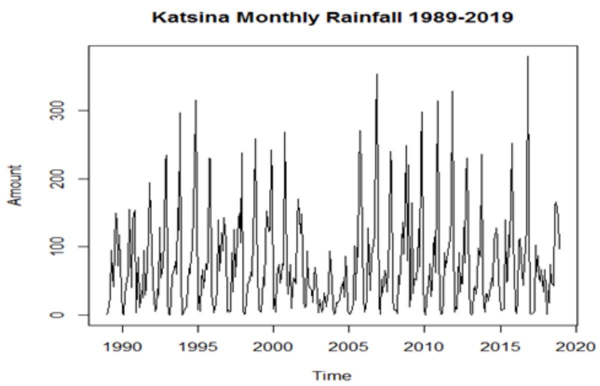

***Figure 4** Katsina monthly rainfall 1989-2019*

Generalized Extreme Value Modeling

This section presents the results extreme rainfall in Katsina using GEV. Table 1 indicates the results of the GEV modeling on extreme rainfall data in Katsina using Block Maxima approach. The GEV parameters were estimated using Maximum Likelihood Estimation (MLE). The rainfall data was fitted in the R software and hence the result. Since the confidence intervals of σ is greater than 0, it implies that the Frechet distribution is the optimal model for the GEV family.

Table1: Generalized Extreme Value Parameter Estimates

Parameters	Location (μ)	Scale (σ)	Shape (ξ)
Estimated Parameter	35.65	37.76	0.39
Standard Error estimated	2.52	2.25	0.07
Confidence interval (95%)	(30.72, 40.59)	(33.34, 42.17)	(0.26, 0.54)

Table 2 shows the predicted maximum rainfall return level (in mm) for the return periods of 2, 20 and 100 years along with their respective 95% confidence intervals. It can be seen from the table that increase in return periods leads to a corresponding increase in return levels. The 100 year return period for the maximum rainfall in Katsina means that, on average it is expected to record a rainfall of 530.28606mm once every 100 years.

Table 2: Generalized Extreme Value Return Level

Return Periods	2years	2years	100 years
Return Level	50.55	249.56	530.29
CI (95 %)	(44.37, 56.73)	(197.16, 301.96)	(331.73, 728.85)

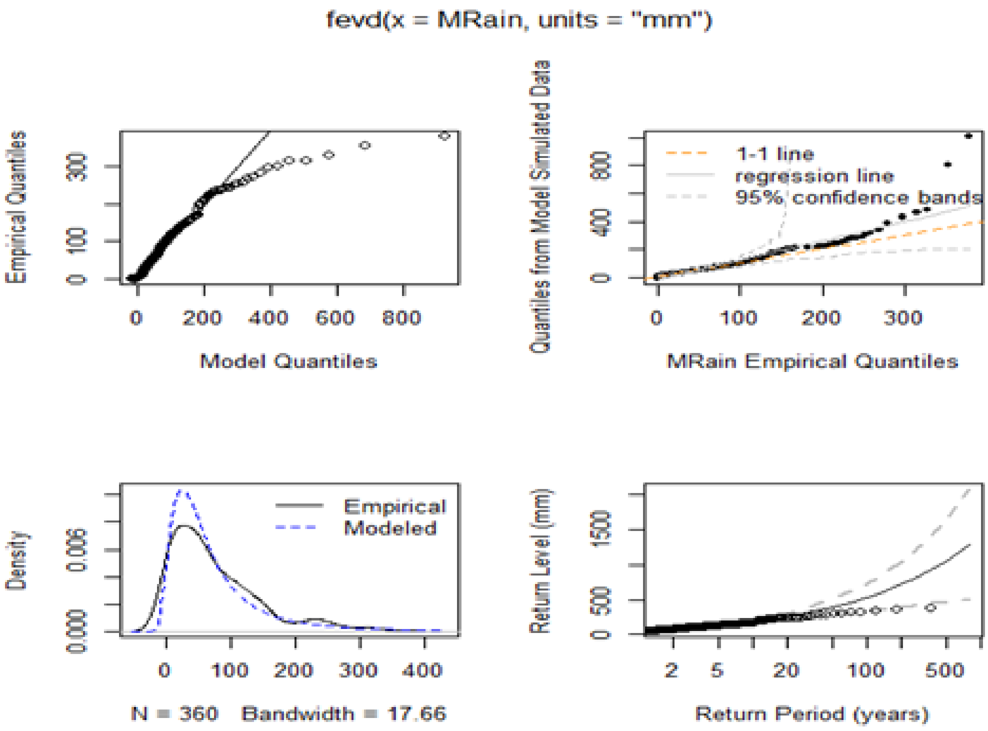

Figure: 5 Generalized extreme value diagnostic and return level plots

Figure 5 shows the diagnostic plots for the GEV. The points of the probability plot and quantile plot lies close to the unit diagonal. This implies that, generalized extreme value distribution function provides a good fit. The return level plot shows that the empirical return levels match well with those from the fitted distribution function. Finally, the density plot also shows good agreement between the fitted GEV distribution function and the empirical density.

Generalized Pareto Distribution modeling

Figure 6 is the mean residual life (MRL) plot for monthly rainfall readings. The mean residual life plot plots average excess value over a given threshold. The threshold is selected from the plot, at mean excess of 83 the index is seen to be higher which gives a threshold u=182.23

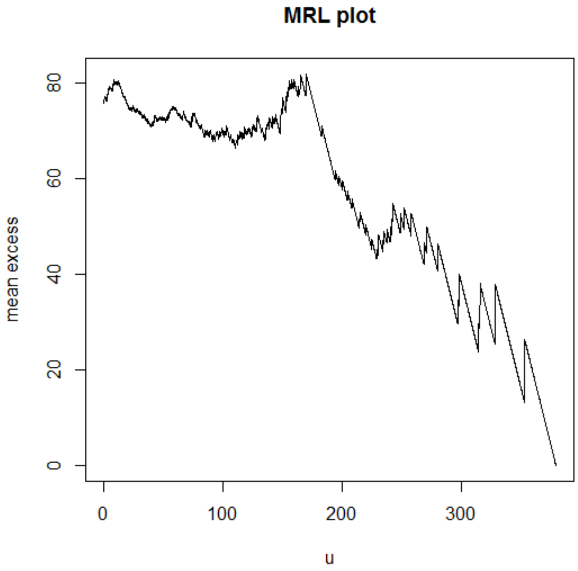

Figure 6 MRL plot Generalized Pareto Distribution modeling

Table 3 shows the parameter estimates of Generalized Pareto Distribution using maximum likelihood estimation. The confidence interval of the scale parameter contains no zero, thus, the distribution belongs to the ordinary Pareto distribution (See 2.3.2).

Table 3: Generalized Pareto Distribution Parameter Estimates

Katsina	**Location (μ)**	**Scale (σ)**
Estimated Parameter	44.70	47.27
Standard error	2.60	2.09
CI (95%)	(6.77, 1.57)	(1.57, 4.39)

Table 4 shows the predicted maximum rainfall return level (in mm) for the return periods of 2, 20, and 100 (in years) along with their respective 95% confidence interval. It can be seen from the results that increase in return periods leads to a corresponding increase in return levels.

Table 4: Generalized Pareto distribution return level

Return Periods	2 years	20 years	100 years
Return Levels	83.57	287.38	593.26
Confidence Interval (95%)	(46.50, 58.27)	(201.16, 302.96)	(372.76, 750.84)

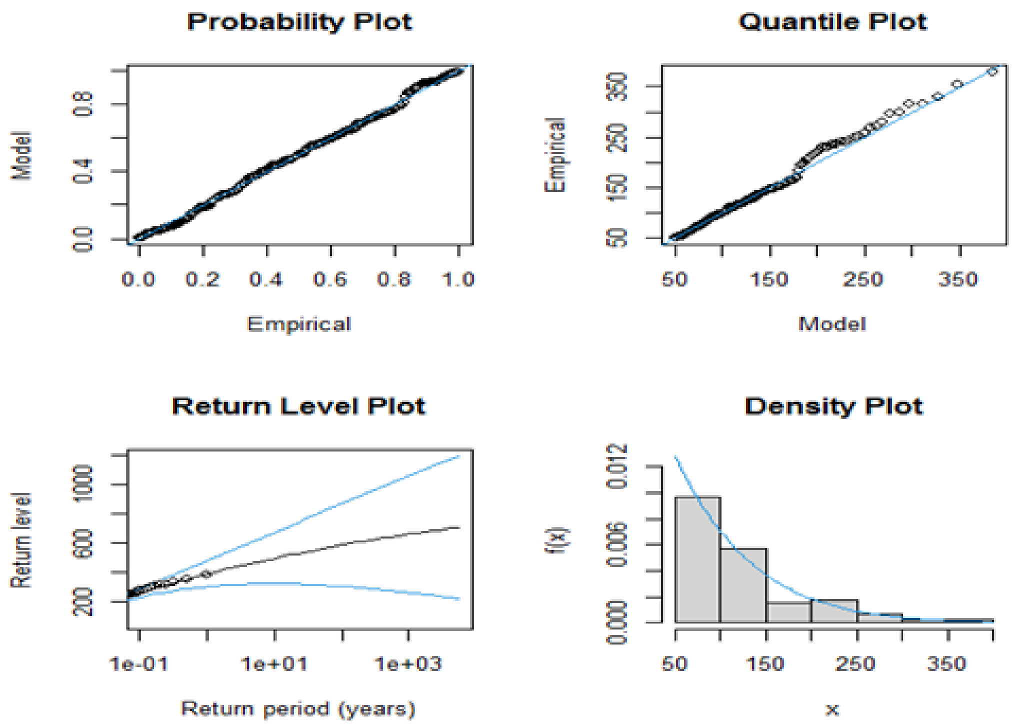

Figure 7 Generalized Pareto distribution diagnostic and return level plots

The model diagnostic plot for GPD looks reasonable for rainfall data in Katsina as it can be seen from figure 7. The points of the probability plot and quantile plot lies close to the unit diagonal. This implies that, GPD provides a good fit. The return level plot shows that the empirical return levels match well with those from the fitted distribution function. Finally, the density plot also shows good agreement between the fitted GPD distributions.

CONCLUSION

This research demonstrates the application and significance of EVT at describing extreme rainfall events in Katsina. The GEVD and GPD models were considered for maximum annual rainfall data in Katsina from 1989 to 2019.The model parameters were estimated using the Maximum Likelihood Estimation, block maxima approach was used to fit the Generalized Extreme Value Distribution while the POT method was used to fit the Generalized Pareto Distribution. Our findings reveal that the Frechet distribution is the optimal model from the GEV family for the annual maximums of monthly rainfall data while the ordinary pareto distribution gave the optimal model for the monthly rainfall data over the threshold. It was found that increase in return periods leads to a corresponding increase in return levels. When comparing the return levels for the GEVD and GPD we observe that the GPD gives higher return levels for 2 and 20years compared to GEVD. However, for higher return periods 100 years it can be seen that the GEVD gives higher return levels compared to the GPD. The model diagnostics showed that the models were reasonable for modeling the rainfall data. This study will help Engineers and decision makers in Katsina with knowledge about extreme rainfall events in the return periods considered, to enable them make appropriate decisions to reduce damage to crops, infrastructure and lives that is caused by extreme rainfall. As climate change remain in existence, adaptation measures and preparedness are essential for the Katsina community's resilience. Thus, this research will be useful in coming up with flood risk early warning, management, preparedness, response and mitigation. It is recommended that adequate measures be taken to stem the trend of irregular rainfall pattern in the years to come. These measures include clearing of drainages and waterways as well as the building of dams to provide water for irrigation and to also help contain water from flash floods as a result of climate change which is very eminent in the northern part of Nigeria. However, future studies can model and predict extreme rainfall in Katsina with respect to specific regions in the state. Also, modeling both extreme rainfall and temperature in Katsina is a possible research direction.

REFERENCES

Anderson, C. W., Coles, S. G. and H¨usler, J.,(1997) "Maxima of Poisson-Like Variables and Related Triangular Arrays", *Annals of Applied Probability,* vol 7. **[Crossref]**

Anderson, C. W.,(1970) "Extreme Value Theory for a Class of Discrete Distributions with Applications to Some Stochastic Processes", *Journal of Applied Probability*, vol 7. **[Crossref]**

Anderson, C. W.,(1980)"Local Limit Theorems for the Maxima of Discrete Random Variables", *Mathematical Proceedings of the Cambridge Philosophical Society*, vol 88. **[Crossref]**

Arnold, B. C. and Villase˜nor, J. A.,(1984) "The distribution of the maximal time till departure from a state in a Markov chain", Statistical Extremes and Applications, NATO Science Series C: *Mathematical and Physical Sciences*, vol 131. **[Crossref]**

Asadi, M., Rao, C. R. and Shanbhag, D. N.(2001), "Some Unified Characterization on Generalized Pareto Distributions", *Journal of Statistical Planning and Inference*, vol 93. **[Crossref]**

Balkema, A. A. and de Haan, L.,(1974)"Residual life time at great age", *Annals of Probability*, vol 2. **[Crossref]**

Behr, R. A., Karson, M. J. and Minor, J. E.,(1991) "Reliability-Analysis of Window Glass Failure Pressure Data", *Structural Safety*, vol 11. **[Crossref]**

Beirlant, J., Goegebeur, Y., Segers, J., and Teugels, J. (2004b). Statistics of Extremes: Theory and Applications. *Probability and Statistics. Wiley.* **[Crossref]**

Broussard, J. P. and Booth, G. G.,(1998) "The Behavior of Extreme Values in Germany's Stock Index Futures: An Application to Intradaily Margin Setting", *European Journal of Operational Research*, vol 104. **[Crossref]**

Brown, B. G. and Katz, R. W., "Regional-Analysis of Temperature Extremes: Spatial Analog for Climate-Change", *Journal of Climate*, vol 8, 1995, pp. 108-119. **[Crossref]**

Burton, Makropoulos, K. C.,(1985)"Seismic Risk of Circum-Pacific Earthquakes: II Extreme Values Using Gumbel's Third Distribution and the Relationship with Strain Energy Release", *Pure and Applied Geophysics*, vol 123. **[Crossref]**

Carlos et al(2002).A Fully Probabilistic Approach to Extreme Value Modelling. Journal of Hydrology, 273, 3550. **[Crossref]**

Castillo, E. and Hadi, A. S.,(1997)"Fitting The Generalized Pareto Distribution to Data", *Journal of the American Statistical Association,* vol 92. **[Crossref]**

Castillo, E., Galambos, J. and Sarabia, J. M.,(1989)"The Selection of the Domain of Attraction of An Extreme Value Distribution from a Set of Data", *Extreme Value Theory, Lecture Notes in Statistics*, vol 5pp. [Crossref]

Chifurira, R., & Chikobvu, D. (2014). Modelling Extreme Maximum Annual Rainfall for Zimbabwe. *In Annual Proceedings of the South African Statistical Association Conference*, 9-16. South African Statistical Association (SASA).

De Haan, L., "On Regular Variation and Its Application to the Weak Convergence of Sample Extremes", *Mathematical Centre Tract*, vol 32, 1970. Amsterdam: Mathematisch Centrum.

de Haan, L.,(1976)"Sample Extremes: An Elementary Introduction", *Statistica Neerlandica*, vol 30. [Crossref]

De Lisle, J. F.,(1965)"Extreme Surface Winds over New Zealand", *New Zealand Journal of Science*,vol 8.

De Waal, D. J.,(1996)"Goodness of fit of the Generalized Extreme Value Distribution based on the Kullback-Leibler Information", *South African Statistical Journal*, vol 30.

Deheuvels, P.,(1986)"Strong laws for the kth Order Statistic when k ≤ c log2 n", *Probability Theory and Related Fields*, vol 72. [Crossref]

ElJabi, N., Ashkar, F. and Hebabi, S.,1998) "Regionalization of Floods in New Brunswick (Canada)",*Stochastic Hydrology and Hydraulics*, vol 12. [Crossref]

Faithful C. O., Alpha B. K., Steeven B. A., Exavery P. E.. &.Mary O. A. (2019). Application of Extreme Value Theory in Predicting Climate Change Induced Extreme Rainfall in Kenya. International Journal of Statistics and Probability; Vol. 8, No. 4. [Crossref]

Ferreira, A., & de Haan, L. (2006). Extreme Value Theory. *Springer*. [Crossref]

Fisher, A.R,& Tippett,L.C(1928). Limiting Forms of The Frequency Distribution of The Largest or Smallest Member of a Sample. *Proceedings of the Cambridge Philosophical society* 24,180-190. [Crossref]

Frechet, M.(1927). Sur la LoI de probabilite de L'Ecart maximum.Annales de la Societe polonaise de mathematique.

Furrer, E.M.,Katz,R.W., Walter,M.D.,& Furrer,R.(2010). Statistical Modeling of Hot Spells and Heat Waves. *Climate Research*. [Crossref]

Galambos, J.,(1982)"A Statistical Test for Extreme Value Distributions", In Nonparametric Statistical Inference.

Gnedenko,B.V.(1943)."On the Limit Distribution of the Maximum term of Random Series "Annals of mathematics.

Goda, Y. (1988). On the methodology of selecting design wave height. In Coastal Engineering 1988 (pp. 899-913).

Goda, Y. (2010). Random seas and design of maritime structures (Vol. 33). World Scientific Publishing Company.

Gumbel, E.J(1941)."The return period of flood flows". The Annals of Mathematical Statistics 12(2). [Crossref]

Gumbel,.E.(1958)."Statistics of Extremes. "Columbia Univ.Pres, New York. [Crossref]

Hanson,L.S.&Vogel,R.(2008).The Probability Distribution of Daily Rainfall in the United States. World Environmental and Water Resources Congress 2008. [Crossref]

Harlow, D. G., Smith, R. L. and Taylor, H. M.,(1983)"Lower Tail Analysis of the Distribution of the Strength of Load-Sharing Systems", *Journal of Applied Probability*, vol 20. [Crossref]

Hasofer, A. M. and Wang, Z.,(1992)"A Test for Extreme Value Domain of Attraction", *Journal of the American Statistical Association*, vol 87. [Crossref]

Jenkinson,A.F.,(1995)."The Frequency Distribution of the Annual Maximum (or Minimum) Values of Meteorological Elements."*Quarterly Journal of the Royal meteorological society*" 81(348). [Crossref]

Katz, R, W. P., and Philippe, N. (2002). Statistics of extremes in hydrology. Advances in water resources, 25(8),1287-1304. [Crossref]

Katz,R.W(2010). Statistics of Extremes in Climate Change. [Crossref]

Kilavi et al (2018): Extreme Rainfall and Flooding over Central Kenya Including Nairobi City during the Long-Rains Season 2018: Causes, Predictability, and Potential for Early Warning and Actions. Atmosphere 2018, 9(12), 472; [Crossref].

Kotz, S. and Nadarajah, S.,(2000)"Extreme Value Distributions: Theory and Applications" London: Imperial College Press. [Crossref]

Kotz, S. and Shanbhag, D. N.,(1980) "Some New Approaches to Probability Distributions", *Advances in Applied Probability*, vol 12. [Crossref]

Lazoglou, G., & Anagnostopoulou, C. (2017). An Overview of Statistical Methods for Studying The Extreme Rainfalls in Mediterranean. In

Multidisciplinary Digital Publishing Institute Proceedings, 1, 681. [Crossref]

Mayooran,T.,& Laheetharan,A.(2014).The Statistical Distribution of Annual Maximum Rainfall in Colombo District. *Srilankan Journal of Applied statistics*,15(2). [Crossref]

Mondal, A., & Mujumdar, P. P. (2015). Modeling Non-Stationarity in Intensity, Duration and Frequency of Extreme Rainfall over India. *Journal of Hydrology*, 521, 217-231. [Crossref]

Nadarajah,S.& Withers,C.S.(2001). Modeling Dependency Between Climate Extremes For New Zealand. World Resource Review 13.

NEMA (2017). 27 states experienced devastating flood disasters tribuneonlineng.com

Pickands, J. (1975). Statistical Inference Using Extreme Order Statistics. The Annals of Statistics, 3(1), 119131. www.jstor.org

Punch Newspaper, Saturday, October 8, 2022. Death, Losses, tears as flood overrun Kogi,Anambra,Gombe, Others pages 22&23. m.facebook.com/punchnewspaper

Ramachandran, G.,(1982) "Properties of Extreme Order Statistics and Their Application to Fire Protection and Insurance Problems", *Fire Safety Journal*, vol 5. [Crossref]

Rao, C. R. and Shanbhag, D. N.,(1994) "Choquet-Deny Type Functional Equations with Applications to Stochastic Models" Chichester: John Wiley and Sons. [Crossref]

Rao, N. M., Rao, P. P. and Kaila, K. L.,(1997)"The First and Third Asymptotic Distributions of Extremes as Applied To The Seismic Source Regions of India and Adjacent Areas", *Geophysical Journal International*, vol 128. [Crossref]

Rosenzweig, C. A. (2001). Climate change and extreme weather events; implications for food production, plant diseases and pests. Global change &human health 2(2): 90-104. [Crossref]

Ryden,J(2010). Statistical analysis of extreme in long-time series and Uppsala. Theoretical and applied climatology. [Crossref]

Suzuki, M. and Ozaka, Y.,(1994) "Seismic risk analysis based on strain-energy accumulation in focal region", Journal of Research of the National Institute of Standards and Technology, vol 99. [Crossref]

Uwimana, O., & Joseph, M. K. (2018). Modelling extreme maximum rainfall using generalized extreme value distribution: Case study kigali city. International Journal of Science and Research (IJSR), 7(6), 121-126.10.21275/ART20183033.

Wang, W., Zhou, W., Fong, S. K., Leong, K. C., Tang, I. M., Chang, S. W., and Leong, W. K. (2015). Extreme rainfall and summer heat waves in macau based on statistical theory of extreme values. Climate Research. [Crossref]

Weilbull,W.(1939)."A statistical theory of strength materials"Vetenskaps academics

Yasuda, T. and Mori, N.,(1997) "Occurrence properties of giant freak waves in sea area around Japan", Journal of Waterway Port Coastal and Ocean Engineering-American Society of Civil Engineers, vol 123. [Crossref]