UMYU Scientifica, Vol. 3 NO. 3, September 2024, Pp 118 – 129
https://doi.org/10.56919/usci.2433.014
ISSN: 2955 – 1145 (print); 2955 – 1153 (online)
A periodical of the Faculty of Natural and Applied Sciences, UMYU, Katsina

ORIGINAL RESEARCH ARTICLE

Modelling and Forecasting Nigeria's Tax Revenue: A Comparative Analysis of SARIMA and Holt-Winters Models

Tasi'u Musa[1], Abdulrazak Usman Moriki[1] and Hussaini Garba Dikko[1]
[1]Department of Statistics, Ahmadu Bello University Zaria, Nigeria

ABSTRACT
Tax revenue is a government's income from taxes imposed on individuals, businesses, and other entities. It is a crucial funding source for public expenditures, such as infrastructure, education, healthcare, security, and social services. Governments rely on accurate tax revenue projections for proper fiscal management and economic planning. According to OECD and IMF reports 2023, tax revenue of Nigeria contributed 44.15% to total expenditure and only 10.86% to the Nation's GDP. This signifies inadequate utilization of appropriate models that may accurately and effectively forecast the tax revenue of Nigeria. This study focused on modelling and forecasting the tax revenue of Nigeria, utilizing SARIMA and Holt-Winters models. Quarterly tax revenue data from January 1990 to December 2022 was used. The Box and Jenkins model identification, estimation, and forecasting procedures were followed accordingly. And results revealed that the SARIMA $(3,2,1)_4\,(0,1,1)_4$ model was selected as the best model among the various models identified based on minimum AIC value. Similarly, the Multiplicative Holt-Winters Model was selected over the additive model on minimum AIC value. The best-fitted models' performance was evaluated using an in-sample forecast with an 80% training set and a 20% validation set, enabling the assessment of forecast accuracy. The results further revealed that the SARIMA model outperformed the Holt-Winters counterpart in forecasting the tax revenue of Nigeria because it minimized the evaluation criteria with an RMSE of 0.1654 and MAE of 0.0816. The study recommended applying the SARIMA model in forecasting the tax revenue of Nigeria.

ARTICLE HISTORY
Received March 05, 2024
Accepted July 22, 2024
Published July 31, 2024

KEYWORDS
Holt-Winters, SARIMA, Forecasting, Tax Revenue, Nigeria

INTRODUCTION

The introduction of personal income tax in Northern Nigeria in 1904 marked the beginning of Nigeria's taxation system, which expanded to the western and eastern regions via the Native Revenue Ordinances in 1917 and 1928, respectively. This system was later integrated into the Direct Taxation Ordinance No. 4 of 1940. Based on British tax legislation of 1948, Nigeria's tax system has seen numerous revisions aimed at enhancement, such as the Federal Inland Revenue Service (Establishment) Act, 2007, and the Companies Income Tax (Amendment) Act, 2007. As described by Sanni (2007), a healthy tax system comprises tax policy, tax law, and tax administration, with laws providing the legal framework and policies guiding administration. The origins of Nigeria's tax laws include laws, the constitution, court rulings, and circulars. Tax enforcement is managed by the Federal, State, and Local Governments, each with distinct jurisdictions as per the Taxes and Levies (Authorized List for Collection) Law of 1998. However, tax collection is largely centralized, with the Federal Government collecting most revenue, including petroleum profit taxes, royalties, company income tax, value-added tax, and customs duties.

The Nigerian tax system's fiscal policy, laws, and administration must work together to achieve economic objectives. As stated by the Presidential Committee on Anons (2008), the primary goal is to improve public policy formulation and efficiently use tax revenue for the populace's benefit. Additional goals include promoting economic development, generating stable revenue for government projects, fostering economic stability, ensuring fairness and distributive equity, and addressing market failures. Tax is a non-refundable regular payment for goods and services that consumers and private businesses often pay to the government. The government is empowered to control and administer tax laws, ensuring all taxes are correctly managed, and revenue is returned to the government.

Otu *et al.,* (2014) applied SARIMA models in modelling and forecasting Nigeria's inflation rates. Utilizing the Box-

Correspondence: Abdulrazak Usman Moriki. Department of Statistics, Ahmadu Bello University Zaria, Nigeria. au.moriki@gmail.com.
How to cite: Tasi'u M., Usman, A. M., & Garba, H. D. (2024). Modelling and Forecasting Nigeria's Tax Revenue: A Comparative Analysis of Sarima and Holt-Winters Models. *UMYU Scientifica*, 3(3), 118 – 129. https://doi.org/10.56919/usci.2433.014

Jenkins methodology, monthly inflation data from November 2003 to October 2013 was used. SARIMA (1, 1, 1) (0, 0, 1)$_{12}$ was selected as the best model for forecasting the monthly inflation rates of Nigeria. Their results revealed that inflation volatility started around 2006, influenced by money supply, exchange rate depreciation, petroleum price hikes, and poor agricultural production.

Etuk & Ojekudo (2015) Utilized the SARIMA model to model and forecast internally generated revenue of Ikot Ekpene Local Government Area in Akwa Ibom State, Nigeria, using internally generated revenue from 1998 to 2007. They introduced an approach that defines subset SARIMA models in terms of autoregressive (AR) models rather than exclusively in moving average (MA) terms. Their findings show that the additive SARIMA model is adequate from the original SARIMA (1,1,0) (1,1,0)$_{12}$ model.

Ajibode (2017) employed the Seasonal Autoregressive Integrated Moving Average (SARIMA) model to forecast enplaned passengers at Muritala Mohammed International Airport using monthly data from 2006-2015. The analysis with the R package revealed non-stationarity and seasonality in the data, requiring differencing to achieve stationarity. The Augmented Dickey Fuller test confirmed stationarity after differencing. The ACF and PACF plots suggested potential models, with the SARIMA (2,1,1)(0,1,1)$_{12}$ model selected based on the corrected Akaike Information Criteria (AICc) for the best fit. Diagnostic tests, including the Ljung-Box and Shapiro tests, indicated that the residuals were independent and normally distributed. This model was used to forecast passenger traffic for 2016-2026, aiding airport and airline planning.

Streimikiene *et al.* (2018) forecasted Pakistan's tax revenue for the fiscal year 2017 using three-time series techniques and analyzed indirect taxes' impact on the working class. The study utilized three different time series models, namely the Autoregressive model (AR with seasonal dummies), Autoregressive Integrated Moving Average model (ARIMA), and Vector Autoregression (VAR) model. The research highlighted the significance of tax analysis and revenue forecasting to guide economic and fiscal policies. Based on the data from July 1985 to December 2016, focusing on forecasting for 2017, the ARIMA model was found to provide the most accurate predictions for total tax revenue in Pakistan. The study revealed that indirect taxes, such as sales tax and customs duties, were major contributors to tax revenue, leading to higher inflation and adversely affecting the working class. Their findings presented the government of Pakistan with valuable insights for policymaking, and the forecasted tax revenue fell short of the government's target.

Samuel & Kibua (2019) studied and forecasted the tax revenue for Kenya using monthly time series data from 2000 to 2015. They adopted the SARIMA method in their study. Evidence from ACF and PACF (as suggested by Box and Jenkins procedure) four SARIMA models were identified: SARIMA (2,0,0) (2,0,0)$_{12}$, SARIMA (2,0,0) (2,1,0)$_{12}$, SARIMA (2,1,0) (2,0,0)$_{12}$, and SARIMA (2,1,0) (2,1,0)$_{12}$. The performance of the model evaluation reveals that SARIMA (2,0,0)(2,1,0)$_{12}$ was the best-fit model for forecasting the Kenyan's Tax Revenue.

Rahmat *et al.* (2020) focused on predicting the allocation of the regional revenue and expenditure budget of APBD (Anggaran Pendapatan dan Belanja Daerah, Indonesia) in the North Sumatra provincial government using time series data from 2002 to 2019. The study utilized the Autoregressive Integrated Moving Average (ARIMA) method for forecasting. Their result shows that the ARIMA method predicted the future value of income and expenditure accounts in the North Sumatra Province Regional Budget. The ARIMA method demonstrated its capability to produce accurate predictions with small standard errors, aiding in the financial planning and execution of regional development in North Sumatra Province.

Kelkar *et al.* (2021) used Seasonal Autoregressive Integrated Moving Average (SARIMA) models to forecast American Southwest Airlines' revenue for 2020. The research identifies SARIMA (0,1,0) (0,1,1)$_4$ as the best-fitted model with the lowest AICs. Diagnostic tests confirm the model's accuracy and a solvency risk analysis is conducted to assess Southwest's financial performance during the COVID-19 pandemic.

Ajisola (2023) analyzed monthly data from the Federal Inland Revenue Service (FIRS) spanning 2010 to 2021, exploring three models for tax revenue forecasting: Multivariate Linear Regression (MLR), Seasonal Autoregressive Integrated Moving Average (SARIMA), and Multivariate Long Short Term Memory Networks (LSTM). The study finds that LSTM and MLR perform well due to their ability to predict using multiple independent variables. LSTM achieves a high R^2 score of 98.9% and an adjusted R^2 score of 98.8%, suggesting its efficacy in forecasting tax revenue.

The study of Rahman *et al.* (2016) analyzed seasonal time series data, specifically the monthly revenue of Bangabandhu Multipurpose Bridge, to forecast revenue using the Holt-Winters method. Both additive and multiplicative seasonal models are considered, and the forecasting accuracy measures of the multiplicative method are found to be higher. However, the research uses the additive Holt-Winters method for forecasting up to January 2021, showing an upward trend in revenue. The forecasted revenue values for specific periods are presented. The coefficient of variation for the monthly revenue and forecasted revenue using the additive Holt-Winters method is determined. The study concludes that the additive Holt-Winters method is superior to the multiplicative method in forecasting the monthly revenue of the bridge.

Fauzi *et al.* (2020) compared the Fuzzy Time Series and Holt-Winter methods for forecasting tourist arrivals from

January 2015 to December 2019. Their findings indicated that Holt-Winter outperforms with a significantly lower Mean Square Error (MSE), making it the preferred forecasting tool. Despite the pandemic's impact, Holt-Winter is chosen for forecasting 2020 and 2021 tourist arrivals.

Ayakeme *et al.* (2021) compared ARIMA and Holt-Winters methods (additive and multiplicative) for forecasting Bayelsa state government's Internally Generated Revenue (IGR) from 2012 to 2018. The ARIMA (0,1,1) model was chosen based on its AIC and BIC values. Forecasting for 2019-2021 was performed, and its accuracy was compared to Winter methods using Mean Absolute Error and Mean Square Error. Their finding reveals that the ARIMA model outperforms Winter methods, recommending its use for future forecasts.

The study of Atoyebi *et al.* (2023) investigated forecasting currencies in circulation (CIC) in Nigeria using the Holt-Winters exponential smoothing methods, both additive and multiplicative. The analysis uses data from January 1960 to December 2022 to determine the optimal forecasting approach and the most effective smoothing parameters. Three sets of parameters were tested, with the combination of $\alpha = 0.4$, $\gamma = 0.3$, and $\delta = 0.1$, providing the best results for both methods. Their results revealed that the multiplicative Holt-Winters method outperformed the additive method in accuracy, with significantly lower MAPE, MAD, and MSD values. The study concluded that the multiplicative Holt-Winters method is superior for forecasting CIC in Nigeria, indicating a continuous upward trend in currency circulation.

Makananisa (2015) utilized SARIMA and Holt-Winters models to model and forecast major taxes, Personnel Income Tax (PIT), Corporate Income Tax (CIT), Value Added Tax (VAT), and Total Tax Revenue (TTAXR) for the South African Revenue Service (SARS). Monthly and quarterly data from 1995 to 2010 are used, and the models are evaluated using various accuracy measures. SARIMA and Holt-Winters models perform well for PIT and VAT, but Holt-Winters outperforms SARIMA for the volatile CIT and TTAXR. The findings demonstrated that the selected models will likely perform well for future forecasts, assuming no significant shocks like an economic recession.

Lwesya & Kibanbila (2017) compared Seasonal ARIMA and Exponential Smoothing models for forecasting tourist arrivals in Tanzania (2000-2009). The study employed a two-staged approach, initially assessing models in similarly structured settings, followed by a comparison in a different setting. Their results indicated that the effectiveness of Seasonal ARIMA (4,1,4) $(3,1,4)_{12}$ and Holt-Winters multiplicative smoothing in a similar setting, but Holt-Winters outperforms Seasonal ARIMA in a different structure.

Thus, from the literature, it was confirmed that time series models were proven to be useful methods for forecasting tax revenue. Therefore, forecasting future revenue with maximum precision is essential for every country's economy. The need for appropriate models that may accurately and effectively forecast the tax revenue for proper policy formulation cannot be overemphasized, as poor models may produce poor and unreliable forecasts and lead to wrong policy formulation, which may eventually result in a collapse of the Nation's economy. According to OECD and IMF reports 2023, tax revenue of Nigeria contributed 44.15% to total expenditure and only 10.86% to the Nation's GDP. This signifies inadequate utilization of appropriate models that may accurately and effectively forecast the tax revenue of Nigeria. Ajie *et al.* (2019) also noted that traditional methods (such as Moving Averages and trend analysis) have limitations in handling seasonality, trend complexities, and outliers in the time series data. Understanding that the SARIMA and Holt-Winters models can trend, seasonality, and outliers in the time series data. This study aims to fill this gap by evaluating the SARIMA and Holt-Winters models, offering insights into their applicability for improving tax revenue forecasts in Nigeria.

The objective of this study is to evaluate and compare the accuracy of SARIMA, and Holt-Winters models in forecasting tax revenue of Nigeria based on forecasting accuracy measures. The study findings also have practical application for the Nigerian government and revenue authorities since precise revenue projections improve fiscal management and act as a guide for budgetary decisions. Researchers and decision-makers interested in enhancing tax revenue forecasting methods will find the study useful.

MATERIALS AND METHOD

Data Description

Secondary data was used for this study. Quarterly tax revenue (petroleum profit tax, company income tax, gas income, capital gains tax, value added tax (VAT), stamp duty, other taxes) data of Nigeria from January 1990 to December 2022 was collected from Federal Inland Revenue Service (FIRS) published on https://www.firs.gov.ng/tax-statistics-report/ to model and forecast tax revenue of Nigeria using Seasonal Autoregressive Integrated Moving Average (SARIMA) and Holt-Winters techniques.

Autoregressive Moving Average Process, ARMA (p, q)

The time series y_t is said to be an ARMA(*p*,*q*) process if it can be represented in the form:

$$y_t = \sum_{j=1}^{p} \phi_j y_{t-1} + \sum_{i=0}^{q} \theta_i \varepsilon_{t-1} \quad (1)$$

But $\theta_0 = 1$

ARMA(p,q) in Lag operator forms

$$y_t = \Theta(L)\varepsilon_t[\Phi(L)]^{-1} \quad (2)$$

Autoregressive Integrated Moving Average, ARIMA (p, d, q)

The mathematical formulation of the ARIMA(p,d,q) model is given below:

$$\Phi(L)\Delta^d y_t = \Theta(L)\varepsilon_t \quad (3)$$

which can be written as

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t \quad (4)$$

where:

a) p, d and q are integers ≥ 0 and refers to the order of Autoregressive Integrated Moving Average parts of the model respectively.
b) d is the level of differencing and is nonnegative integer. Generally, d = 1 is enough is most cases. Note, when d = 0 then it reduces to an ARMA (p, q) model.
   ARIMA (p,0,q) = ARMA(p, q).
c) ARIMA (p,0,0) is the AR(p) model and ARIMA(0,0,q) is the MA(q) model.
d) ARIMA (0,1,0) i.e $y_t = y_{t-1} + \varepsilon_t$ is a special model called random walk. It is widely used for non-stationary data, such as stock price and exchange rate series.

Seasonal ARIMA (p,d,q) $(P,D,Q)_s$

A first order seasonal difference is the difference between an observation and the corresponding observation from the previous year, and is calculated as:

$$\Delta^s y_t = y_t - y_{t-s} \quad (5)$$

For monthly time series s = 12, for quarterly time series s = 4 and so on. This model is generally termed as SARIMA (p,d,q) $(P,D,Q)_s$ model. The mathematical formulation of the model in terms of lag polynomials is given as:

$$\Phi(L^s)\phi(L)(1-L^s)^D(1-L)^d y_t = \Theta(L^s)\theta(L)\varepsilon_t \quad (6)$$

which can be written as

$$\Phi(L^s)\phi(L)\Delta^{sD}\Delta^d y_t = \Theta(L^s)\theta(L)\varepsilon_t \quad (7)$$

where Δ^{sD} is the seasonal difference series

d = Number of non-seasonal differencing

D = Number of seasonal differencing

s = Seasonal period

ε_t : is unobserved white noise series with mean zero and variance σ^2.

AR part:

$$\Phi(L^s) = 1 - \Phi L^s - \Phi_2 L^{2s} - .... - \Phi_p L^{ps}$$ (Seasonal part) (8)

MA part:

$$\Theta(L^s) = 1 + \Theta L^s - \Theta_2 L^{2s} - .... - \Theta_Q L^{Qs}$$ (Seasonal part) (9)

Holt Winters Model

The Holt-Winters model is designed for a time series that exhibits linear, trend and seasonality. This comprises two methods which include the additive Holt-Winters method and multiplicative Holt-Winters method.

Additive Holt-Winters Method

The additive Holt-Winters method is appropriate when a time series has a linear trend with an additive seasonal pattern, for which the level, the growth rate and the seasonal pattern may be changing. The additive model smoothing equations are given by:

$$l_t = \alpha(y_t - S_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1}) \quad (10)$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \quad (11)$$

$$s_t = \gamma(y_t - l_t) + (1-\gamma)S_{t-m} \quad (12)$$

Multiplicative Holt-Winters Method

If a time series has a linear trend with a fixed growth rate b and a fixed seasonal pattern s_t with increasing variation. The multiplicative model smoothing equations are given by:

$$l_t = \alpha\left(\frac{y_t}{s_{t-m}}\right) + (1-\alpha)(l_{t-1} + b_{t-1}) \quad (13)$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1} \quad (14)$$

$$s_t = \gamma\left(\frac{y_t}{l_t}\right) + (1-\gamma)s_{t-m} \quad (15)$$

where (from equation 3.15 to 3.20)

y_t is the observed value at time t,

l_t is the level (intercept) at time t,

b_t is the trend (slope) at time t,

s_t is the seasonal component at time t,

m is the number of seasons in a cycle

$\alpha, \beta, and \gamma$ are smoothing parameters for level, trend, and seasonality, respectively.

Modelling and Forecasting

Box and Jenkins procedures and Holt-Winters methods were employed to model and forecast tax revenue of Nigeria using SARIMA and Holt-Winters models.

(1) Model Identification

Plot the time series data and choose proper transformation. Through careful examination of the fitted plot get an idea on whether the series contains trend, seasonality, outliers, nonconstant variance or is generally non-stationary. Then, compute and examine the ACF and the PACF of the original series to further confirm a necessary degree of differencing so that the differenced series is stationary. In this case, if the ACF decays very slowly and the PACF cuts off at lag 1, then taking the first difference $(1-L)y_t$ is adequate. Alternatively, to achieve stationarity of the series, we performed a unit root test as proposed by Dickey and Fuller (1979) to remove non-stationarity of the series.

Unit Root Test

ADF test H_o : The data is not stationary.

H_1: The data is stationary.

Decision rule: Reject H_o if p-value is less than alpha (0.05) otherwise fails to reject.

After we achieved the stationarity of the series we then tested for seasonality in the series.

Friedman Rank test

Under this test; H_o : There is no seasonality in the series.

H_1: There is seasonality in the series.

Decision rule: If the p-value (obtained from the test) is less than 0.05, reject H_o otherwise fail to reject.

at α = 0.05

Having removed non-stationarity and seasonality in the series, we compute and examine the ACF and PACF of the properly transformed and differenced series to identify the orders of p and q of the various SARIMA models.

Similarly, we visualized the data for the Holt-Winters models' identification to understand its structure, trends, and seasonality. The series was decomposed into trend, seasonal, and residual components. Then, we computed initial values for level, trend, and seasonal components. This was done using simple averages or other heuristics to choose the Holt-Winters models.

(2) Model Parameter Estimation

At this point, the focus is on determining which model coefficient values best fit the data. To provide estimators that maximize the likelihood of observing the given data under a specified model, making them more efficient and statistically sound. The Maximum Likelihood Estimation (MLE) method was employed in this study. After stationarity was attained and seasonality was removed in the series, the information criteria for estimating models' parameters was utilized to conclude which model explains the series better by comparing Akaike's criterion Criterion (AIC) for each model (SARIMA and Holt-Winters). The model with the smallest AIC value is considered the best model.

The Akaike Information Criteria (AIC) is given by:

$$AIC(p,q) = -2\ln(L) + 2k \tag{16}$$

where

L is the maximized value of the likelihood function for the estimated model.

k is the number of parameters in the model.

(3) Model Diagnostics Checking

Here, the diagnostics checks were conducted to assess the adequacy of the fitted model. This entails determining if the estimated parameters are statistically significant and whether the residuals exhibit a white noise process. The ACF and PACF plots of residuals, and the Ljung-Box tests were inspected to verify that the residuals from the model are sufficient (random).

Ljung-Box test

The Ljung-Box test statistic is based on the sum of squared autocorrelations of the residuals at different lags. The formula for the test statistic is given by:

$$LB = n(n+2)\sum_{k=1}^{h}\frac{\hat{\rho}_k^2}{n-k} \tag{17}$$

where:

n is the sample size.

h is the number of lags being tested.

$\hat{\rho}_k^2$ is the sample autocorrelation at lag k

This statistic follows a chi-square (χ^2) distribution with degrees of freedom equal to number of lags being tested (h).

Under this test; H_o : There is no significant autocorrelation in the residuals.

H_1: There is significant autocorrelation in the residuals.

Decision rule: If $Q > \chi^2_{\alpha,h}$, reject H_o otherwise fail to reject.

at $\alpha = 0.05$

(4) Forecasting

When the adequate models were identified, forecasts for future time periods were generated using the fitted SARIMA and Holt-Winters models. We employed in-sample forecast to predict the values within the dataset used for model training. This method enabled us to assess a model's ability to capture historical patterns, revealing its effectiveness in replicating past trends and behaviors. However, to avoid the risk of overfitting, where the model may perform well on existing data but struggle to generalize to new observations, we complement in-sample forecasting with out-of-sample testing to ensure the model's robustness and accurate predictions on unseen data.

(5) Forecast Evaluation

The accuracy measures of the forecasts were assessed using appropriate evaluation metrics such as Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) where the best forecasting model is considered based on minimum of these statistics. These accuracy measures are represented mathematically as follows:

a. Root Mean Square Error is given by

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_t - \hat{y}_t)^2} \quad (18)$$

b. Mean Absolute Error is given by

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}\frac{|y_t - \hat{y}_t|}{\hat{y}_t} \quad (19)$$

RESULTS AND DISCUSSION

This study employs the SARIMA model and Holt-Winters methods, including both additive and multiplicative approaches, to assess the tax revenue of Nigeria spanning January 1990 to December 2022. The study aims to analyze and forecast the patterns of Nigeria's tax revenue over the specified period using time series forecasting techniques.

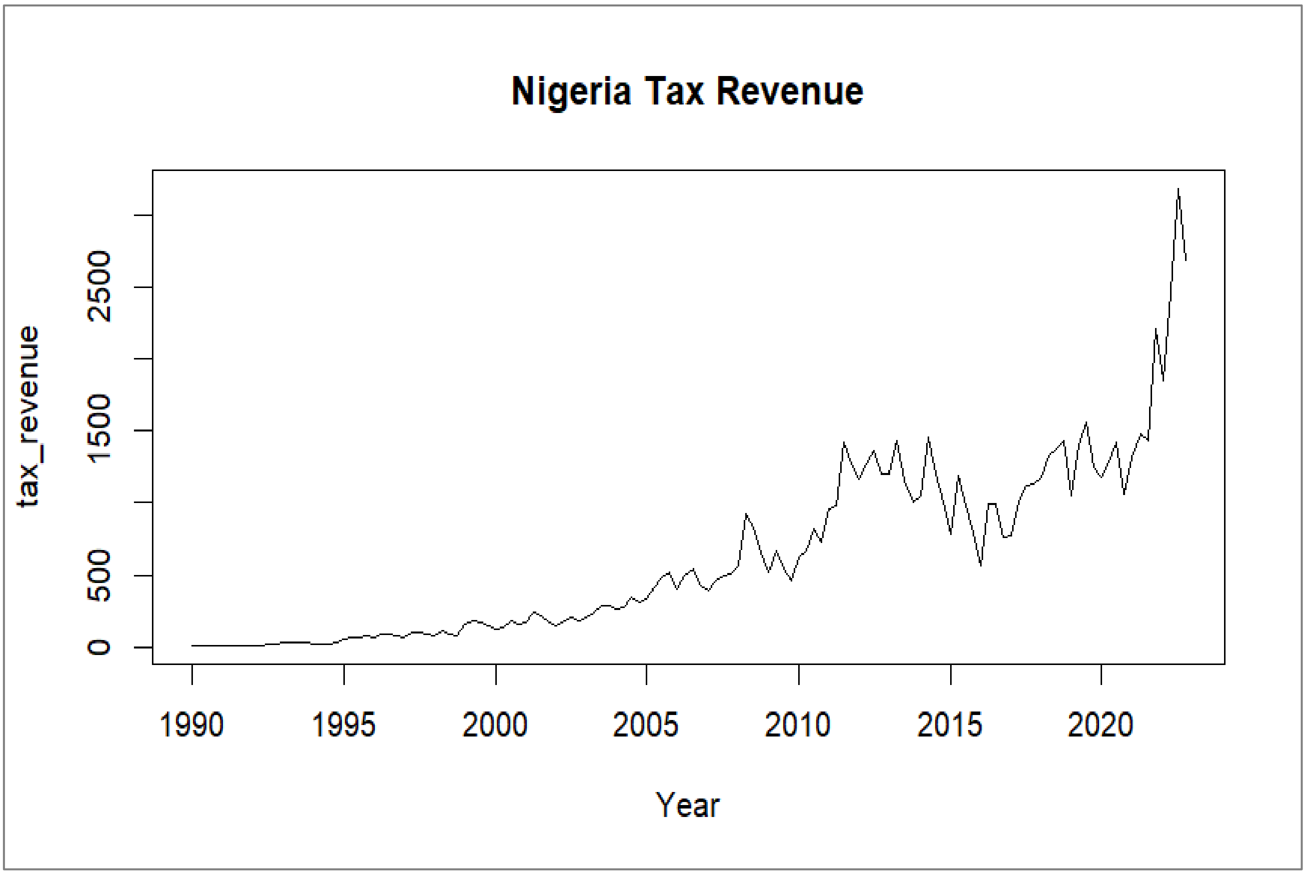

Figure 1: Time plot of the original tax revenue data

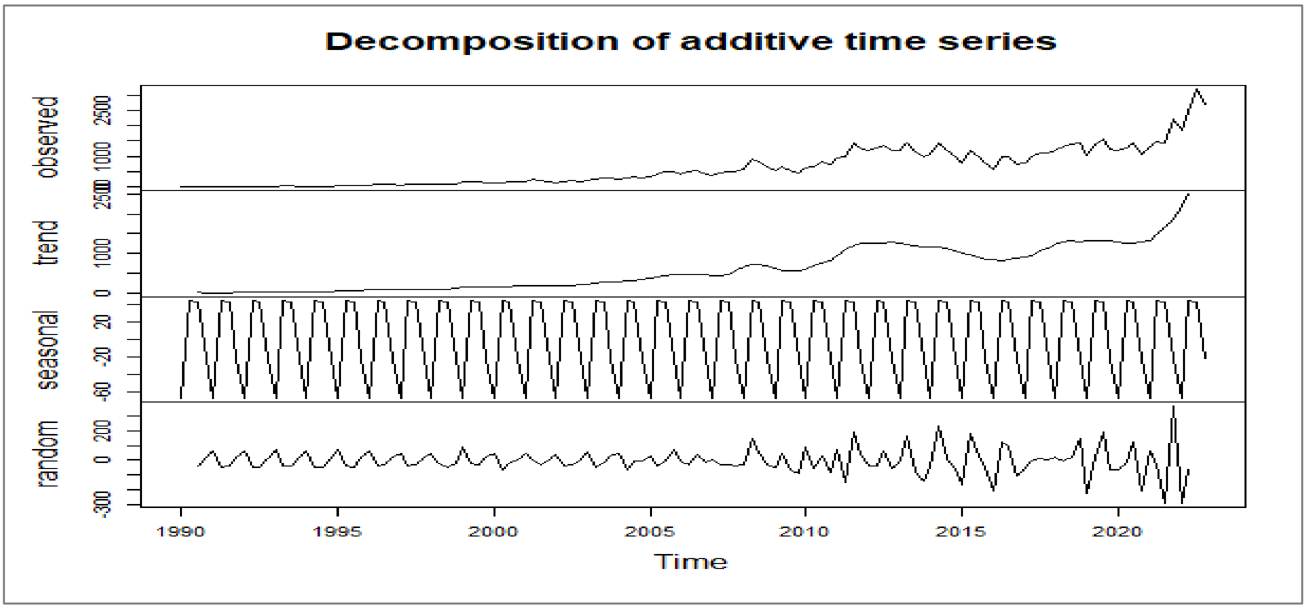

Figure 2: Decomposition of the original tax revenue data

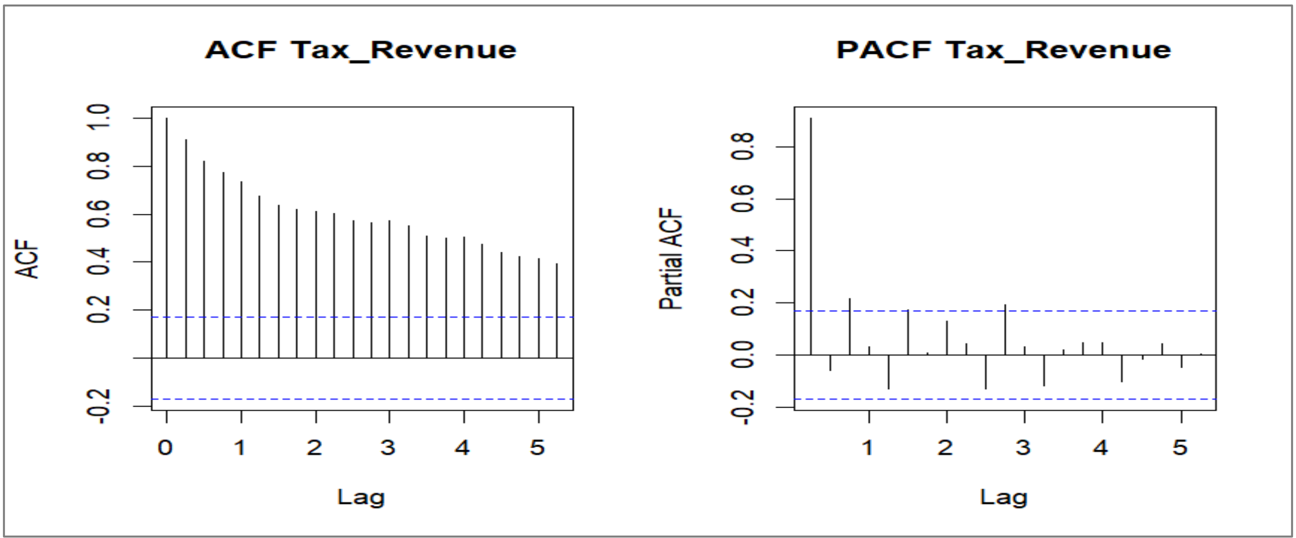

Figure 3: ACF and PACF plots of the original tax revenue data

The tax revenue data time plot for Nigeria from January 1990 to December 2022 is shown in Figure 1. The time plot shows trends and fluctuations over time. This indicates that the tax revenue data is non-stationary, in which the data transformation and differencing are required to establish the stationarity of the series. The analysis of the tax revenue data decomposition presented in Figure 2 provides evidence of trend, seasonality, and random effects in the time series data, with a particularly strong seasonal component. Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots of tax revenue data are shown in Figure 3. Outside the interval boundaries, a damped sine wave can be seen in the ACF and PACF plots. This indicates that the tax revenue data is not stationary at this level. So, to achieve stationarity, the series was subjected to a stationary/unit root test to further confirm the non-stationary tax revenue data as indicated in both the time series plot and ACF and PACF plots of the original data.

Augmented Dicky-Fuller (ADF) Unit Root Test for the (Non-Seasonal)

The stationarity of the data is an essential condition to proceed further and employ any time series model. The method most frequently employed to check for stationarity in a data series is the Augmented Dickey–Fuller (1979). Results from ADF tests at level first and second differencing were displayed in Table 1.

Table 1: Results of Augmented Dickey-Fuller Unit Root Test

Test Statistic	At level	At 1st differencing	At 2nd differencing
ADF statistic	-1.9031	-2.3001	-5.6424
p-value	0.6168	0.4517	0.01

Under the ADF test, the null hypothesis is "the data is not stationary." Table 1 indicates that at the level and first differencing, the p-values of the ADF test are 0.06168 and 0.4517 respectively, which is greater than 0.05; therefore, we fail to reject the null hypothesis. Moreover, at the second differencing, the p-value of the ADF test is 0.01 which is less than 0.05 significant level. This shows enough evidence to reject the null hypothesis, and we conclude that the time series is stationary after the second differencing.

Seasonality test

Detecting and removing seasonality is crucial for accurate time series modeling and forecasting. Here, we applied periodogram analysis, and Friedman rank to identify seasonality in time series data.

Friedman Rank Test

Table 2: Results of Friedman Rank Test for Seasonality

Test Statistic	P-value
32.74	3.658563e-07

Under the Friedman rank test, the null hypothesis is "there is no seasonality in the series." Table 2 indicates that the p-value of Friedman rank test is 0.01, which is less than 0.05 significant level. This shows that there is enough evidence to reject the null hypothesis, and we conclude that there is seasonality in the series.

Removing Unit Root by Differencing (Non-Seasonal)

Having confirmed that the tax revenue data is not stationary, this is a crucial step in preprocessing time series data, particularly when fitting SARIMA and Holt-Winters models. By carefully applying and evaluating differencing to ensure the models are built on stationary data, leading to more accurate and reliable forecasts.

Figure 4 shows the stabilization of the mean of the time series, and changes in the level were removed in the series. This made the series suitable for modelling and forecasting the tax revenue of Nigeria using SARIMA and Holt-Winters models.

Removing Seasonality in the series by seasonal differencing

Seasonal differencing is essential to handle time series data with regular seasonal patterns. Removing seasonality transforms the series into a stationary one, facilitating more accurate modeling and forecasting. This process is especially important for the SARIMA technique, which requires the input series to be stationary. Through careful application and validation, seasonal differencing enhanced the effectiveness of time series modelling and forecasting.

Figure 5 indicates the mean of time series was stabilized, and seasonal trends were eliminated. This made the series more reliable for modelling and forecasting the tax revenue of Nigeria using SARIMA and Holt-Winters models.

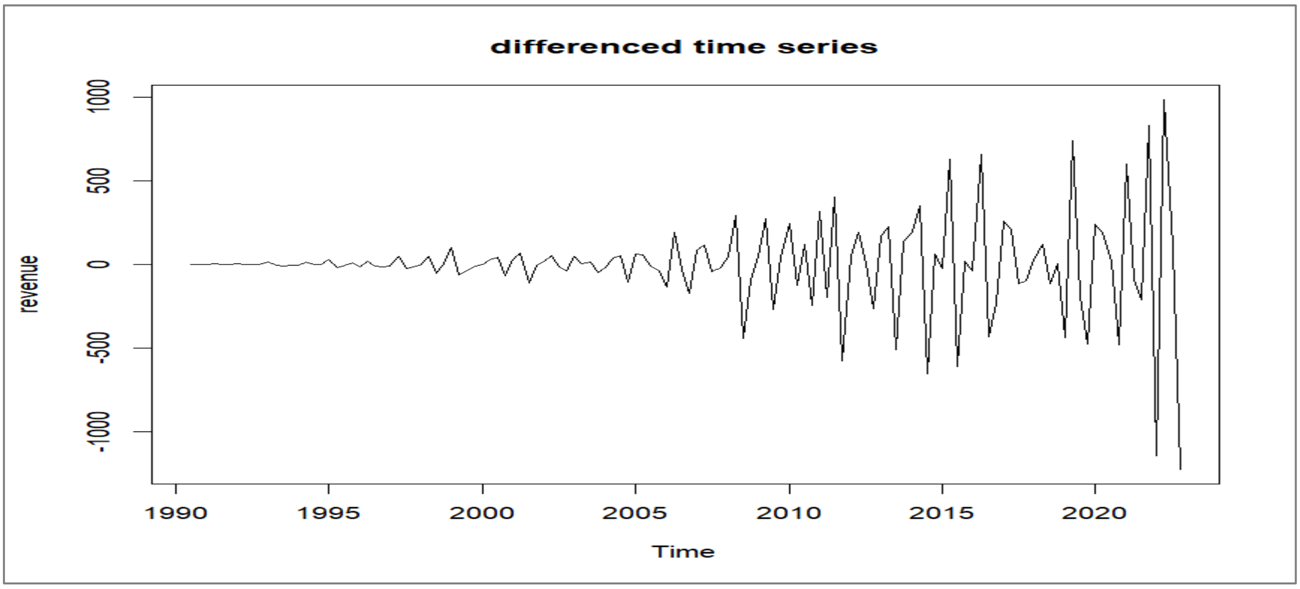

Figure 4: Differenced series (non-seasonal)

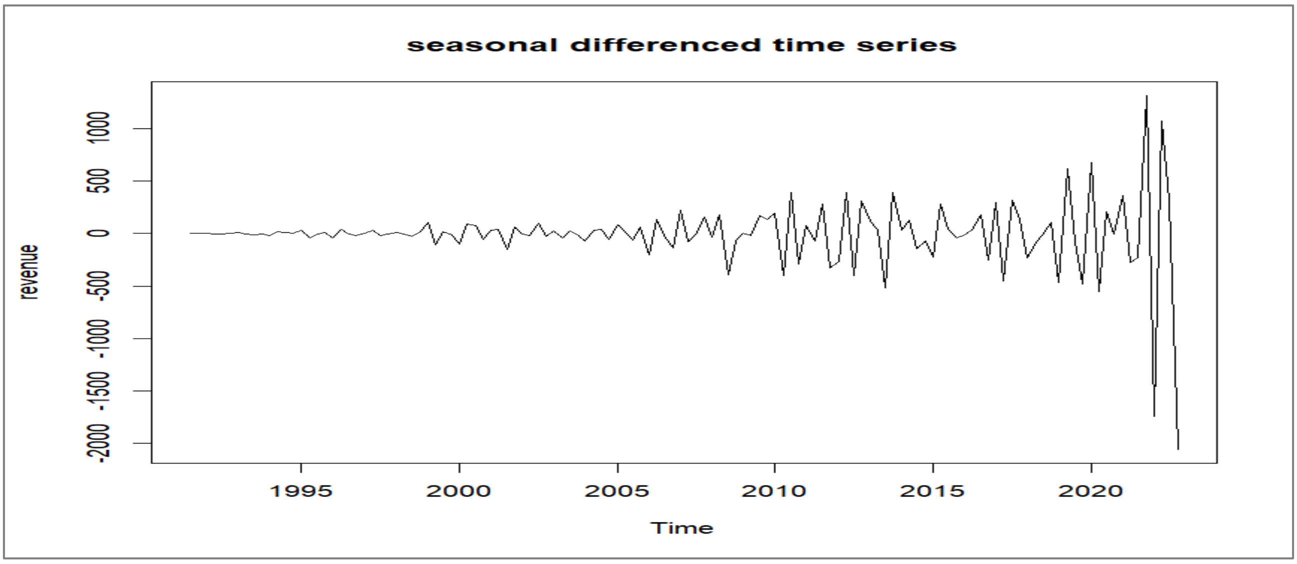

Figure 5: Seasonal differenced series plot

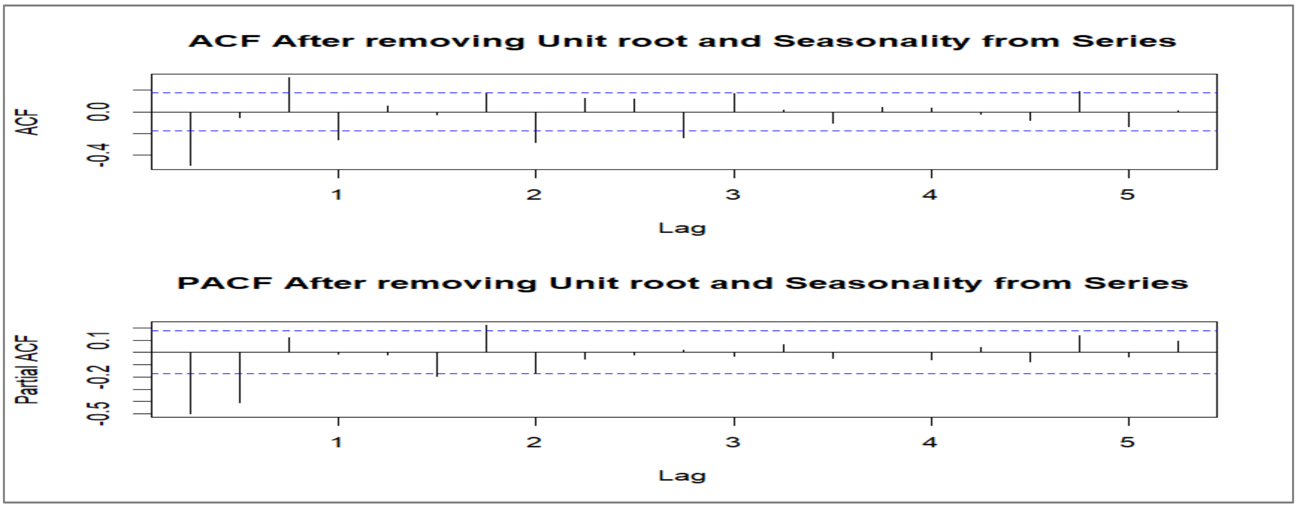

Figure 6: ACF and PACF plots of the Seasonal differenced series

Models Selection for Optimal Fit

The choice of SARIMA and Holt-Winters models involved utilizing the Akaike Information Criterion (AIC) to compare various suggested models. The optimal models deemed the best fit for the data were selected based on the results presented in Tables 2 and 3, respectively.

Table 3 shows the various identified SARIMA models using ACF and PACF with their Akaike Information Criteria (AIC) values. The SARIMA (3,2,1) $(0,1,1)_4$ model was found to be the best fit to the data based on the AIC values and was selected as the best-fitted model for forecasting tax revenue of Nigeria.

The Holt-Winters methods and Akaike Information Criteria (AIC) were applied to identify the best-fitted model between Additive and Multiplicative Holt-Winters methods. In which the Multiplicative Holt-Winters method, having the lowest AIC values, was selected as the best-fitted model to forecast the tax revenue of Nigeria, as shown in Table 4.

Parameter Estimation

In this stage, the parameters of the best models of SARIMA and Holt-Winters models were estimated using the maximum likelihood method to determine the appropriateness of the models, as shown in Tables 5 and 6.

Table 5 shows the estimated parameters of best fitted SARIMA (3,2,1) $(0,1,1)_4$ model. The p-values of the estimated parameters are all less than the significant level of 0.05. Hence, all the parameters of the best-fitted model are statistically significant. Therefore, the SARIMA (3,2,1) $(0,1,1)_4$ model is appropriate for forecasting tax revenue of Nigeria.

Table 6 indicated that the p-values of the estimated smoothing parameters of the multiplicative Holt-Winters model are all lower than the significant level of 0.005.

Therefore, all the parameters of the best-fitted model are statistically significant, and the selected Multiplicative Holt-Winters model is suitable for forecasting the tax revenue of Nigeria.

Models Adequacy Checks

Under the Ljung-Box test, the null hypothesis is "there is no autocorrelation in the residuals". Table 7 shows that the Q statistic (6.974) is lower than $\chi^2_{\alpha,h}$ (16.750) at 0.05 significant level. Therefore, we fail to reject H_0 and we conclude that there is no autocorrelation in the residuals. The proposed SARIMA (3,2,1) $(0,1,1)_4$ is adequate and satisfies all model assumptions.

Similarly, the null hypothesis is "there is no autocorrelation in the residuals." Table 8 shows that the Q statistic (36.826) is lower than $\chi^2_{\alpha,h}$ (39.997) at 0.05 significant level. Therefore, we fail to reject H_0 and we conclude that there is no autocorrelation in the residuals. The proposed multiplicative Holt-Winters model is sufficient and satisfies all model assumptions.

Forecast Performance Measures

The accuracy measures were used to assess the precision and effectiveness of a forecasting model. These measures provided insights into how well the models predicted the future values compared to actual observations, as indicated in Table 9.

The performance of the best-fitted models was evaluated using an in-sample forecast. We used 80% of the actual data (a training set) to build the models, and the remaining 20% of the actual data was used as a validation set (i.e., we used the built model to forecast the remaining 20% of the data. This process enabled us to see and assess the performance of the models. In terms of producing accurate or nearly accurate forecasts (minimizing the forecast error). Conventionally, researchers are using MSE, RMSE, MAE and so on to assess the performance of a given model compared with others. A model with a minimum value of these measures is regarded as best. These measures are computed and presented in Table 9.

Table 3: SARIMA Model Selection

Model	AIC
SARIMA (3,2,1) $(0,1,1)_4$	**16.4652**
SARIMA (3,2,1) $(1,1,2)_4$	16.4746
SARIMA (3,2,1) $(0,1,2)_4$	16.4748
SARIMA (3,2,1) $(1,1,1)_4$	16.4795
SARIMA (2,2,1) $(0,1,1)_4$	16.4836
SARIMA (3,2,2) $(0,1,1)_4$	16.4838
SARIMA (2,2,2) $(0,1,1)_4$	16.4925
SARIMA (2,2,1) $(0,1,1)_4$	16.5003
SARIMA (2,2,1) $(0,1,2)_4$	16.5017
SARIMA (2,2,1) $(1,1,1)_4$	16.5029
SARIMA (2,2,2) $(0,1,1)_4$	16.5082
SARIMA (2,2,1) $(0,1,2)_4$	16.5184
SARIMA (2,2,1) $(1,1,1)_4$	16.5198
SARIMA (2,2,1) $(1,1,2)_4$	16.5232
SARIMA (2,2,2) $(0,1,2)_4$	16.5262
SARIMA (2,2,2) $(1,1,2)_4$	16.5267
SARIMA (2,2,2) $(1,1,1)_4$	16.5289
SARIMA (2,2,1) $(1,1,2)_4$	16.5409
SARIMA (3,2,0) $(0,1,1)_4$	16.5456
SARIMA (1,2,1) $(0,1,1)_4$	16.5625

Table 4: Holt-Winters Model Selection

Model	AIC
Holt-Winters's Additive Model	187.8944
Holt-Winters's Multiplicative Model	**24.84788**

Table 5: SARIMA (3,2,1) (0,1,1)$_4$ Model Parameters Estimation

Parameter	Coefficient	Standard Error	t-statistic	p-value
ϕ_1	-0.9331	0.0872	-10.7037	0.0001
ϕ_2	-0.7899	0.1039	-7.6008	0.0001
ϕ_3	-0.4595	0.1070	-4.2934	0.0001
θ_1	-0.9999	0.0697	-1.1014	0.0001
Θ_1	0.3478	0.1391	2.5013	0.0131

Table 6: Multiplicative Holt-Winters Model Smoothing Parameters Estimation

Smoothing Parameters	Coefficient	Standard Error	t-statistic	p-value
α	0.530	0.075	7.068	< 0.001
β	0.299	0.122	2.451	< 0.001
γ	0.290	0.012	24.167	< 0.001

Table 7: Ljung-Box test for SARIMA (3,2,1) (0,1,1)$_4$

Q statistic	$\chi^2_{\alpha,h}$	Degree of freedom
6.974	16.750	5

Table 8: Ljung-Box test for multiplicative Holt-Winters Model

Q statistic	$\chi^2_{\alpha,h}$	Degree of freedom
36.826	39.997	20

Table 9: Summary of Accuracy Measures of the Models

	Forecasting Models	RMSE	MAE
SARIMA Model	**SARIMA (3,2,1) (0,1,1)$_4$**	**0.1654**	**0.0816**
Holt-Winters Model	Multiplicative Holt-Winters model	0.1767	0.1096

CONCLUSION

This study focused on modelling and forecasting the tax revenue of Nigeria, utilizing SARIMA and Holt-Winters models. Quarterly tax revenue data from January 1990 to December 2022 was used. The Box and Jenkins model identification, estimation, and forecasting procedures were followed accordingly. The results revealed that the SARIMA (3,2,1) (0,1,1)$_4$ model was the best among the various models identified based on minimum AIC value. Similarly, the Multiplicative Holt-Winters Model was selected over the additive model on minimum AIC value. The best-fitted models' performance was evaluated using an in-sample forecast with an 80% training set and a 20% validation set, enabling the assessment of forecast accuracy. The results further revealed that the SARIMA model outperformed the Holt-Winters counterpart in forecasting the tax revenue of Nigeria because it minimized the evaluation criteria with an RMSE of 0.1654 and MAE of 0.0816. The study recommends adopting the SARIMA model for more accurate tax revenue forecasting in Nigeria. This model's demonstrated accuracy can aid in better fiscal planning and policy development.

As the study employed SARIMA and Holt-Winters models to model and forecast tax revenue of Nigeria, these models operate under certain assumptions that may not fully capture the complex nature of tax revenue fluctuations. These models do not account for external factors such as economic policies and global economic conditions, which are unpredictable and can influence tax revenue forecasting. Having highlighted the identified limitations, the study suggested using advanced techniques such as Regime-Switching models, Kalman Filter and machine learning techniques to have a more robust and accurate forecast of Nigeria's tax revenue.

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