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ORIGINAL RESEARCH ARTICLE

Mathematical Modeling of Kidnapping: Dynamics and Control

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ABSTRACT
Kidnapping is a major societal issue. This study develops a mathematical model categorizing kidnappers by their activities. The model is divided into six compartments: susceptible individuals (S), informers (I), kidnappers (K), kidnappers soldiers (K_S), , kidnappers leaders (K_L), and individuals undergoing rehabilitation (R). The model assesses control strategies by calculating the effective reproduction number. A reproduction number less than 1 indicates the potential eradication of kidnapping activities, while a higher number suggests persistence. The results reveal that eliminating as well as incarcerating kidnappers and their leaders can significantly reduce kidnapping incidents.

INTRODUCTION

Kidnapping is a significant societal challenge with far-reaching consequences for individuals, communities, and governments. Understanding the dynamics of kidnapping incidents is crucial for devising effective prevention and intervention strategies.

Kidnapping is a criminal offense that occurs when a person abducts another person without his consent for the main purpose of financial ransom, political objectives, or other nefarious objectives. Inyang and Abraham (2013). Kidnappings affect some African countries. This is a national issue that has to be rectified. However, a great number of people have been abducted and kidnapped by criminals for a variety of reasons and purposes, such as rape, illegal sexual relations, selling human parts for ritual sacrifice, political retaliation, slavery, ransom-bargaining, marriage, murder or assassination, unlawful activities, and other reasons. Aniayam et al. (2018).

The current level of insecurity has severely hampered the country's security architecture, so more information is needed to address the recent surge in violent criminal attacks in Nigeria. The combination of terrorist and banditry attacks in the country has not only put a significant financial strain on fiscal policy but has also compelled an increase in the amount of money the national government spends on security. Rufai (2021). Along with cattle rustling, kidnapping and kidnapping for ransom were embraced as new tactics. Many impoverished young people from many communities served as informants, providing valuable intelligence to large financial interests. Umejei (2010). According to a summary of the statistics, Abia State led the group with 110 kidnapping cases overall, 58,109 arrests, 41 prosecutions, and one fatality.

Akwa Ibom registered 40 kidnapping instances, 418 arrests, 11 prosecutions, and 43 releases. Delta recovered 44 kidnapping cases, 27 arrests, 31 prosecutions, and one death. The study also stated that kidnappers stole over 600 million dollars between July and September 2008 and July 2009. Also Zamfara over 5000 kidnapping cases were recorded. But beyond statistics being available, it is a known fact that most kidnapping cases are never reported to the police authority for fear of murdering the victims; hence, most families prefer to pay ransom to lose one of their own. Dodo et al. (2008).

Lawal et al. (2023) also presented a mathematical model for halting the growth of armed banditry in Nigeria. Five ordinary differential equations make up the model, which accounts for the dynamics of bandits B(t), recoveries R(t), informants I(t), and susceptible people S(t) and E(t). The authors examine the effectiveness of two time-dependent controls in reducing the demographic profile of informants and bandits by incorporating them into the model: measures to make armed banditry unprofitable and the establishment of jobs. The authors characterize the optimal control model using the Pontryagin Maximum Principle (PMP) and simulate using the Forward-Backward Sweep Method of the fourth-order Runge-Kutta scheme.

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Kidnapping is one of the major social problems, like cultism, violence, and banditry. Mathematicians such as (Adamu and Ibrahim (2020) and Okoye et al. (2021) have contributed toward eradicating the problem through mathematical modeling, but their models did not incorporate classifying kidnappers by their roles. Hence, this work will present a mathematical model classifying kidnappers by their roles and activities, offering a nuanced representation of the phenomenon.

MODEL FORMULATION:

The population is divided into six compartments: susceptible individuals (S(t)), informers (I(t)), kidnappers (K(t)), kidnappers sponsors, kidnappers leaders, and individuals undergoing rehabilitation (R(t)). Every compartment symbolizes a different set of individuals who play particular roles and exhibit particular behaviors associated with kidnapping.

The model incorporates various parameters, including recruitment rates (ʌ), death rates (δ), transmission rates between compartments, and probabilities of specific events such as informers becoming kidnappers or kidnappers becoming leaders, δ is the death due to kidnappers activities, death due to torture and life jail in rehabilitation, rate at which susceptible becomes informers, rate at which informers becomes kidnappers. μ natural death rates, ψ rate at which kidnappers become kidnappers sponsors, t is the fraction at which kidnappers become military leaders, (1-t) is the probability that not all kidnappers become leaders, σ rate at which all individuals fall back to K(t) from R(t), β is a rate at which kidnappers sponsors moved to rehabilitation, ∝ movement from rehabilitation back to susceptible. τ is jail break due to kidnappers operations; *e* is recruitment from susceptible to kidnappers. The schematic diagram is represented in Figure 1 below.

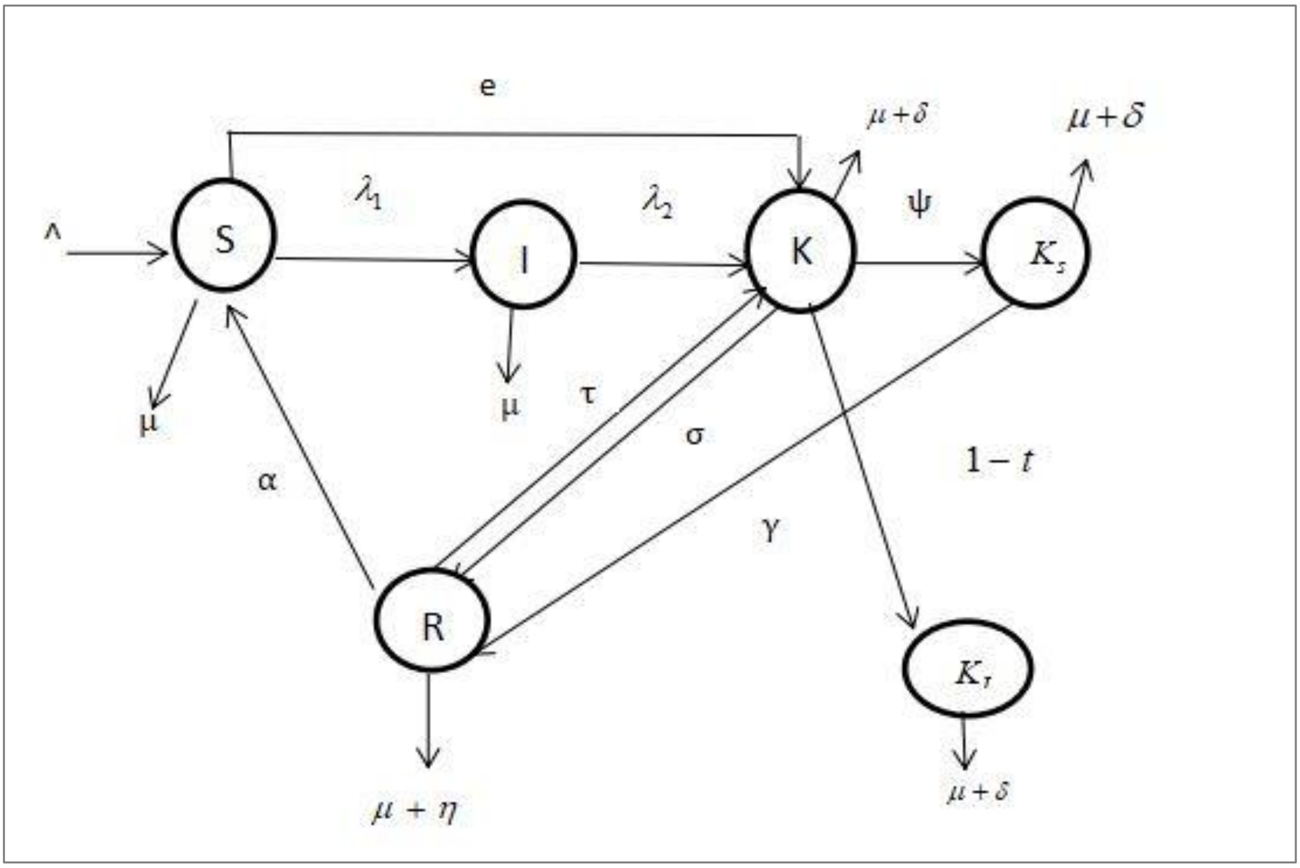

Figure 1: Flow diagram of the kidnapping network.

The corresponding mathematical representation of the model is given by following differential equations.

$$\frac{dS}{dt} = \Lambda + \alpha R - (eS + \mu + \lambda_1)S \quad (.1)$$

$$\frac{dI}{dt} = \lambda_1 S - \mu I - \lambda_2 I \quad (.2)$$

$$\frac{dK}{dt} = \lambda_2 I + eS + \sigma R - (\tau + \psi)K - (1-t)K - (\mu + \delta)K \quad (3)$$

$$\frac{dK_S}{dt} = \psi K - \gamma K_S - (\mu + \delta) K_S \qquad (.4)$$

$$\frac{dK_L}{dt} = (1 - t) K - (\mu + \delta) K_L \qquad (5)$$

$$\frac{dR}{dt} = \tau K + \gamma K_S - \sigma R - \alpha R - (\mu + \eta) R \quad . \qquad (6)$$

$$where, \quad \lambda_1 = \frac{\beta_1 K}{N} \quad and \quad \lambda_2 = \frac{\beta_2 K}{N} \quad . \qquad (7)$$

BASIC PROPERTIES OF THE MODEL EQUATIONS

1 Invariant Region: The population size can be determined by using the Nonlinear differential equation of the model formulated.

$$N = S + I + K + K_S + K_L + R \qquad (8)$$

Equation (8) resolved to a Nonlinear differential equation of the form:

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dK}{dt} + \frac{dK_S}{dt} + \frac{dK_L}{dt} + \frac{dR}{dt} \qquad (9)$$

i.e

$$\frac{dN}{dt} = \Lambda - \mu (S + I + K + K_S + K_L + R) - (K_S + K)\delta - \eta R \qquad (10)$$

Such that

$$\frac{dN}{dt} \leq \Lambda - \mu N \qquad (11)$$

Theorem 2.: The solution of the system of equations (1) to (6) is feasible for t< 0 if they enter the invariant area Ω.

Proof: suppose $(S, I, K, K_S, K_L, R) \in R^6$ be any solution of the system with a non-negative initial condition. By method of integrating factor:

$$\text{I.F} = e^{\int p dt} = e^{\mu t} + c = e^{\mu t} . e^c = A e^{\mu t}$$

$$A e^{\mu t} \frac{dN}{dt} = \int A e^{\mu t} \Lambda \qquad (12)$$

$$N(t) = \frac{\Lambda}{\mu} + C e^{-\mu t} \quad . \qquad (13)$$

At t =0, the initial population will become

$$N(0) = \frac{\Lambda}{\mu} + C \quad , \qquad (14)$$

Where C is constant

Simplifying we've.

$$C = N_0 - \frac{\Lambda}{\mu}, \quad (15)$$

by substitution

$$N(t) = \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu}\right)e^{-\mu t} \quad (16)$$

$$N(0) \le \frac{\Lambda}{\mu}. \quad (17)$$

at $t \to \infty$ in equation (13), the human population N approaches $C = \frac{\Lambda}{\mu}$, i.e., N → C,

Where $C = \frac{\Lambda}{\mu}$ is the caring capacity.

Hence, all feasible solution set of the population of the model system in (1) to (6) entered the region.

$$\Omega = \left(S, I, K, K_{S,} K_L, R \in R^6{}_+ : S, I, K, K_{S,} K_L, R \ge O \therefore N \le \frac{\Lambda}{\mu}\right). \quad (18)$$

Hence its positively invariant set under the induced by the model (1) to (6); hence the model is mathematically well-posed in the domain.

Table 1: Description of variables and parameters.

Variables/Parameters	Descriptions
S(t)	Susceptible individuals
I(t)	Informers
K(t)	Kidnappers
$K_S(t)$	Kidnappers sponsors
$K_L(t)$	Kidnappers leaders
R(t)	Individuals undergoing Rehabilitation
N(t)	Total population
Λ	Recruitment rate(birth or immigration)
λ_1	rate at which susceptible are recruited to informers
λ_2	the conversion rate from informant to kidnappers
Ψ	Progression rate of kidnappers to kidnappers sponsor
τ	Jail break from R back to susceptible .
(1-t)	the probability that not all kidnapper become leader.
μ	Natural death rate
η	death due to torture / life jail in rehabilitation.
δ	Death rate due to kidnappers activities.
α	Progression rate of individuals under rehabilitation back to susceptible
σ	Rehabilitation rate of kidnappers
γ	rate at which kidnappers sponsors move to rehabilitation.

EXISTENCE AND POSITIVITY OF SOLUTIONS.

Here the following result guarantee by the kidnappers model governed in equation (1) is well–posed in a feasible region Ω.

Lemma: Let ∈ Ω be the starting condition. Hence, if t >0, the model's solution set from equations (1) to (6) is positive.

Proof: from equation (1)

$$\frac{dS}{dt}=\Lambda+\alpha R-(e+\mu+\lambda_1)S \tag{19}$$

$$\frac{dS}{dt}\geq-(e+\mu+\lambda_1)S$$

Using separation of variable we have,

$$\frac{dS}{S}\geq-(e+\mu+\lambda_1)dt$$

Integrating both sides to have,

$$\ln(S)\geq-(e+\mu+\lambda_1)t+c$$

$$S(t)\geq ce^{-(e+\mu+\lambda_1)t}.$$

$$at\ \ t=o,$$

$$S(0)\geq C_1.$$

Likewise, it is evident that S(t), I(t), K(t), KS(t), KL(t), and R(t) > 0

EXISTENCE OF EQUILIBRIUM E^*

At this point, every variable's rate of change is zero.

$I^*=0, K^*=0, K_S{}^*=0,\ K_L{}^*=0,$ and by solving the model equations yields to:

$$E_0=\left(\frac{\Lambda}{\mu},\ 0,\ 0,0,0,0\right). \tag{20}$$

EXISTENCE OF KIDNAPPING–FREE EQUILIBRIUM AND EFFECTIVE REPRODUCTION NUMBER.

Diekmann and Heesterbeek's (2010) and Feng & Huang's (2002) concepts on models (1) to (6) were used to develop the stability of the kidnapping-free equilibrium. Let Fi(x) represent the rate at which a new infection appears in a compartment, and let Vi(x) represent the rate at which a new infection decreases in a compartment due to infection flow within the system of infected compartments.

$$F=\begin{pmatrix}\frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial k_S} & \frac{\partial f_1}{\partial k_L}\\ \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial k_S} & \frac{\partial f_2}{\partial k_L}\\ \frac{df_3}{\partial I} & \frac{df_3}{\partial k} & \frac{df_3}{\partial k_S} & \frac{df_3}{\partial k_L}\\ \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial k} & \frac{\partial f_4}{\partial k_S} & \frac{\partial f_4}{\partial k_L}\end{pmatrix}=\begin{pmatrix}\frac{\lambda_1\Lambda}{\mu} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}, \tag{21}$$

Also,

$$V = \begin{pmatrix} \frac{\partial v_1}{\partial I} & \frac{\partial v_1}{\partial k} & \frac{\partial v_1}{\partial k_S} & \frac{\partial v_1}{\partial k_L} \\ \frac{\partial v_2}{\partial I} & \frac{\partial v_2}{\partial k} & \frac{\partial v_2}{\partial k_S} & \frac{\partial v_2}{\partial k_L} \\ \frac{dv_3}{\partial I} & \frac{dv_3}{\partial k} & \frac{dv_3}{\partial k_S} & \frac{dv_3}{\partial k_L} \\ \frac{\partial v_4}{\partial I} & \frac{\partial v_4}{\partial k} & \frac{\partial v_4}{\partial k_S} & \frac{\partial v_4}{\partial k_L} \end{pmatrix} = \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 \\ 0 & \psi & k_2 & 0 \\ 0 & k_4 & 0 & k_3 \end{pmatrix}. \quad (22)$$

where,

$$k_1 = (\tau + \psi + 1 - t + \mu + \delta) \qquad k_4 = (1-t)$$
$$k_2 = (\gamma + \mu + \delta) \qquad k_5 = (\sigma + \sigma + \mu + \eta)$$
$$k_3 = (\mu + \delta).$$

Therefore,

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu} & 0 & 0 & 0 \\ 0 & \frac{1}{k_1} & 0 & 0 \\ 0 & \frac{-\psi}{k_1 k_2} & \frac{1}{k_2} & 0 \\ 0 & \frac{-k_4}{k_1 k_3} & 0 & \frac{1}{k_3} \end{pmatrix}. \quad (23)$$

Multiplying equations (21) and (23) will give

$$\left(FV^{-1}\right) = \begin{pmatrix} \frac{\lambda_1 \Lambda}{\mu^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (24)$$

$$\left|FV^{-1} - \lambda I\right| = \begin{pmatrix} \frac{\lambda_1 \Lambda}{\mu^2} - \lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{pmatrix}, \quad (25)$$

$$therefore \quad Rc = \frac{\beta_1 \Lambda}{N\mu^2}$$. which is the spectral eigen value of (24) (26)

EXISTENCE OF LOCAL STABILITY OF KIDNAPPING FREE-EQUILIBRIUM.

Theorem: the kidnapping-free equilibrium point is locally asymptotically stable if $R_C<1$ and unstable if $R_C>1$.

Proof. We use the Jacobian stability technique as in Adamu & Ibrahim (2020)

$$J(E_0) = \begin{pmatrix} e+\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 \\ e & 0 & k_1 & 0 & 0 & 0 \\ 0 & 0 & \psi & -k_2 & 0 & 0 \\ 0 & 0 & k_4 & 0 & -k_3 & 0 \\ 0 & 0 & \tau & \gamma & 0 & -k_5 \end{pmatrix}. \tag{27}$$

$$J(E_0 - \lambda I) = \begin{pmatrix} e+\mu-\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu-\lambda & 0 & 0 & 0 & 0 \\ e & 0 & k_1-\lambda & 0 & 0 & 0 \\ 0 & 0 & \psi & -(k_2+\lambda) & 0 & 0 \\ 0 & 0 & k_4 & 0 & -(k_3+\lambda) & 0 \\ 0 & 0 & \tau & \gamma & 0 & -(k_5+\lambda) \end{pmatrix} \tag{28}$$

The determinant gives,

$$\left(-e\lambda + ek_1 - \lambda_1 e + \lambda^2 - \lambda\mu - \lambda k_1 + \mu k_1\right)(\mu-\lambda)(-k_2-\lambda)(-k_3-\lambda)(-k_5-\lambda) . \tag{29}$$

Simplifying equation (29) gives

$$\lambda^6 + (-e - 2\mu - K_1 + K_2 + K_3 + K_5)\lambda^5 + (eK_1 - \lambda_1 e + \mu K_1 - (-e - \mu - K_1)\mu$$
$$-(e + 2\mu + K_1)K_2 + (-e - 2\mu - K_1 + K_2)K_3 - (e + 2\mu + K_1 + K_2 - K_3)K_5)\lambda^4$$
$$+\left(-(eK_1 - \lambda_1 e + \mu K_1)\mu - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2\right) + (eK_1 - \lambda_1 e$$
$$+\mu K_1 - (-e - \mu - K_1)\mu - (e + 2\mu + K_1)K_2)K_3 - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu$$
$$+(-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2 - (eK_1 - \lambda_1 e + \mu K_1 - (-e - \mu - K_1)\mu - (e + 2\mu + K_1)K_2)$$
$$+(e + 2\mu + K_1)K_2 - (-e - 2\mu - K_1 + K_2)K_3)K_5)\lambda^3 + \left(-(eK_1 - \lambda_1 e + \mu K_1)\mu K_2\right)$$
$$+(-(eK_1 - \lambda_1 e + \mu K_1)\mu - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2)K_3 - ((eK_1 - \lambda_1 e - \mu K_1)\mu$$
$$K_3)K_5)\lambda^2 + (-(eK_1 - \lambda_1 e + \mu K_1)\mu K_2 K_3 - ((eK_1 - \lambda_1 e + \mu K_1)\mu K_2 - (-(eK_1 - \lambda_1 e + \mu K_1)\mu - ($$
$$-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2)K_3)K_5)\lambda - (eK_1 - \lambda_1 e + \mu K_1)\mu K_2 K_3 K_5 .$$

Collect the coefficient of the eigenvalues λ and characteristic gives,

$$\lambda^6 + w_5\lambda^5 + w_4\lambda^4 + w_3\lambda^3 + w_2\lambda^2 + w_1\lambda + w_0 = 0 . \tag{30}$$

Where

$$w^6 = 1,$$

$$w_5 = \left(-e - 2\mu - K_1 + K_2 + K_3 + K_5\right),$$

$$w_4 = \left(eK_1 - \lambda_1 e + \mu K_1 - (-e - \mu K_1)\mu - (e + 2\mu + K_1)K_2 + (-e - 2\mu - K_1 + K_2)K_3 - (e + 2\mu + K_1 + K_2 - K_3)K_5\right),$$

$$W3 = +\mu K_1 - (-e - \mu - K_1)\mu - (e + 2\mu + K_1)K_2)K_3 - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu + (e + 2\mu + K_1)K_2 - (-e - 2\mu - K_1 + K_2)K_3)K_5).$$

$$w_2 = \left(-(eK_1 - \lambda_1 e + \mu K_1)\mu K_2\right) + (-(eK_1 - \lambda_1 e + \mu K_1)\mu - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2)K_3 - ((eK_1 - \lambda_1 e - \mu K_1)\mu K_3)K_5).$$

$$w_1 = (-(eK_1 - \lambda_1 e + \mu K_1)\mu K_2 K_3 - ((eK_1 - \lambda_1 e + \mu K_1)\mu K_2 - (-(eK_1 - \lambda_1 e + \mu K_1)\mu - (-eK_1 + \lambda_1 e - \mu K_1 + (-e - \mu - K_1)\mu)K_2)K_3)K_5)$$

$$w_0 = -(eK_1 - \lambda_1 e + \mu K_1)\mu K_2 K_3 K_5.$$

The Routh-Hurwitz criterion is used to show that all the eigenvalues have negative real parts. As a result, the kidnapping-free equilibrium is locally asymptotically stable if and only if equation (30) is satisfied and there are no kidnappers.

$$w_1 > 0,\ w_2 > 0,\ w_3 > 0,\ w_4 > 0,\ w_5 > 0, w_6 > 0,\ w_1 w_2 w_3 > w_0 {w_3}^2 - {w_1}^2 w_4 > 0, \tag{31}$$

$$\begin{pmatrix} w_1 > 0,\ w_2 > 0,\ w_3 > 0,\ w_4 > 0,\ w_5 > 0, w_6 > 0,\ w_1 w_2 w_3 > w_0 {w_3}^2 - {w_1}^2 w_4 > 0, \\ (w_1 w_5 - w_0 w_6)(w_1 w_2 w_3 - {w_4}^2 - w_0 {w_1}^2 w_5) > 0,\quad w_5 (w_1 w_2 - w_3)^2 + w_0 w_1 {w_5}^2 > 0, \\ w_6 (w_2 w_3 w_4 - w_5)^3 + w_0 w_1 w_2 {w_6}^3 > 0. \end{pmatrix} \tag{32}$$

Hence, the kidnapping-free equilibrium is LAS whenever $R_C<1$.

EXISTENCE OF ENDEMIC EQUILIBRIUM.

The population's endemic equilibrium point is a positive steady-state solution where abduction activity continues. Consequently, there is an endemic equilibrium point (E**) in the model equations (1) through (6). The endemic equilibrium of equations (1) through (6) is stable asymptotically locally because there are kidnappers. Nevertheless, the recruitment of kidnappers will not decrease when $R_C > 1$.

PARAMETER ESTIMATION.

Parameter estimation and sensitivity analysis are crucial steps in mathematical epidemiology to ensure that the models accurately reflect the dynamics of real-world disease spread. These iterative processes of parameter estimation and sensitivity analysis are essential for refining epidemiological models and improving their predictive capabilities. They help ensure that the models capture the complexities of disease transmission dynamics and provide valuable insights for decision-making

Table 2: Values for variables of the model

S/N	variables	Hypothetical values	Source
1	S	3000000	Assumed
2	I	700000	Assumed
3	K	15000	Assumed
4	K_S	20000	Assumed
5	K_L	5000	Assumed
6	R	30000	Assumed
7	N	3,770,000	calculated

Table .3 Values for parameters of the model

S/N	Parameter	Parameter value	Source
1.	Λ	6000	Assumed
2	λ_1	0.0006	Assumed
3	λ_2	0.0007	Assumed
4	μ	0.017	Adamu & Ibrahim(2020)
5	E	0.00001	Assumed
6	Ψ	0.8	Assumed
7	δ	0.003	Assumed
8	η	0.0035	Assumed
9	T	0.0007	Assumed
9	α, γ, σ	0-1	Controls

NUMERICAL SIMULATIONS.

The following initial conditions and variables were used in the simulations: t = 5 years was the last time.

The rate Λ = 6000 Assumed, λ_1 = 0.0006, λ_2 = 0.0007, μ = 0.017 Calculated, e = 0.00001 Assumed, τ = 0.0007 Assumed, δ = 0.0003 hypothetical, η = 0.0035 Assumed, ψ = 0.0008 Assumed, $(\alpha = \gamma = \sigma = 0 - 1)$ Controls, S(t) = 300000 hypothetical, I(t) = 700000 Assumed, K(t) = 1500000 hypothetical, $K_L(t)$ = 20000 Assumed, $K_S(t)$ = 5000 Assumed, R(t) = 30000 Assumed, N(t) = 1205000 Calculated. It is shown in Tables (2) and (3) respectively.

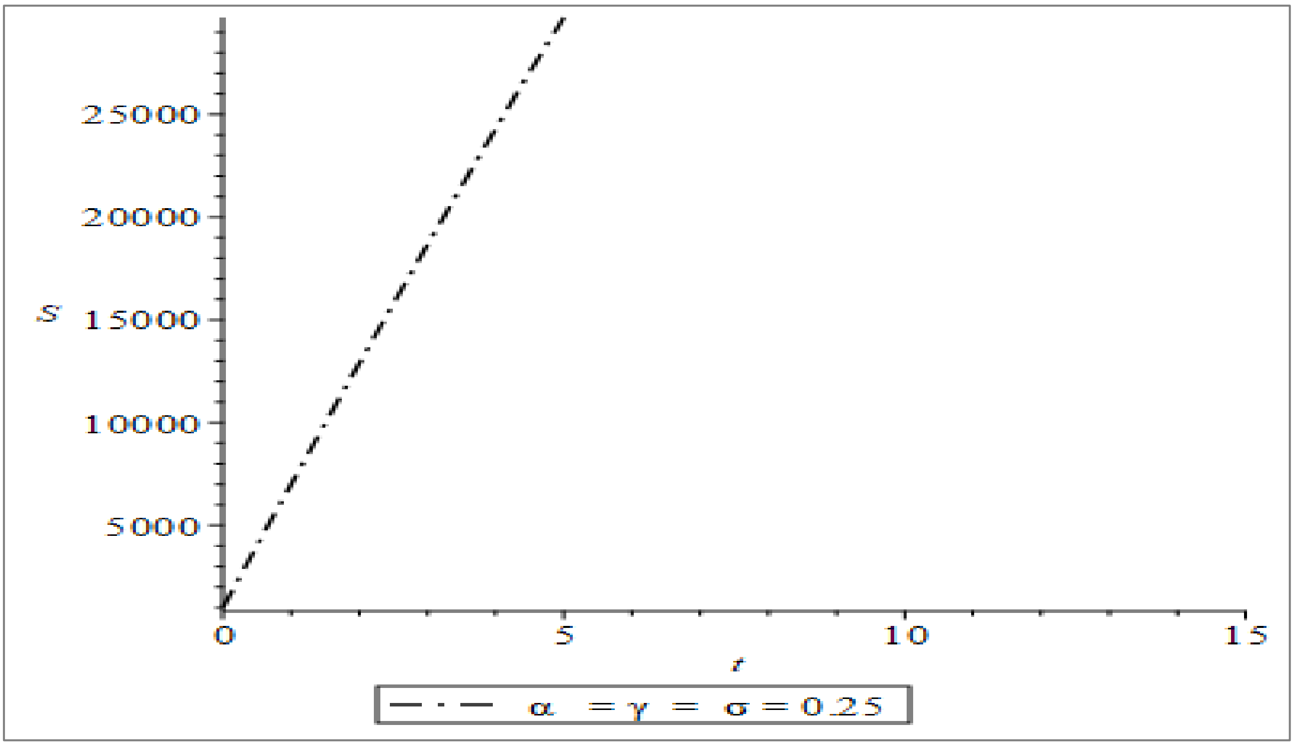

Figure 2: Graph of susceptibility with control:

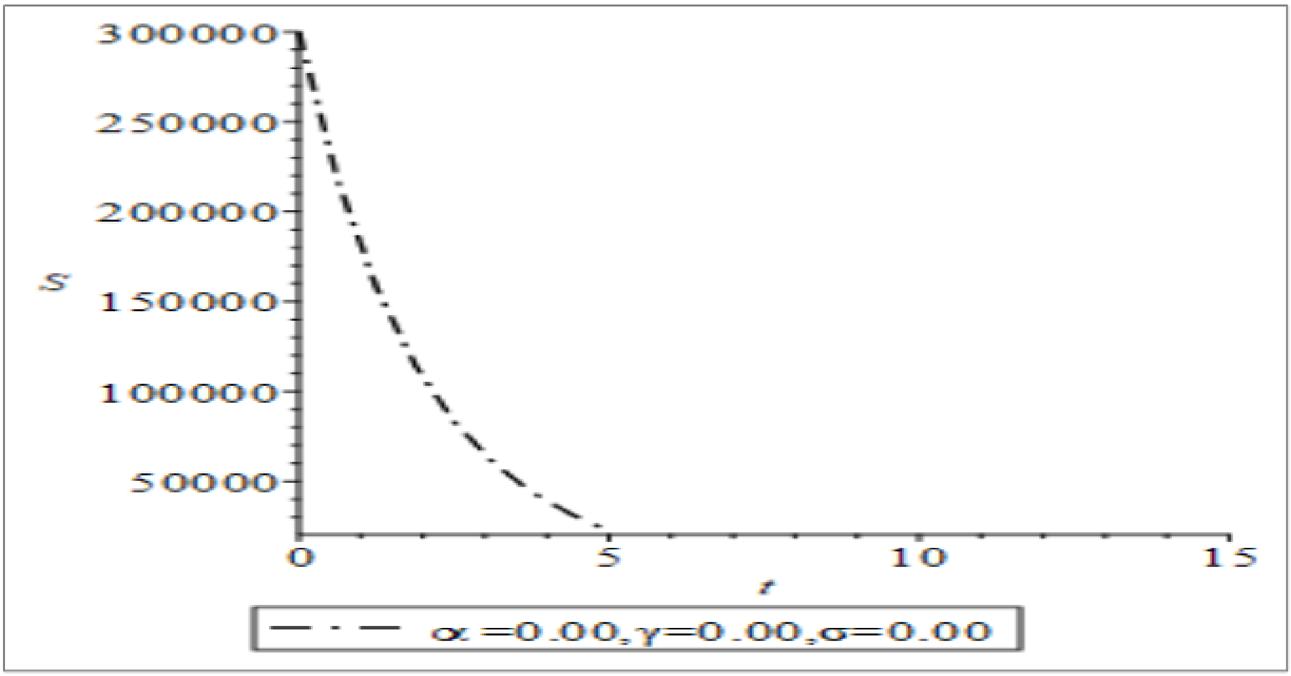

Figure 3: Graph of susceptibility with absence of control:

Figure 2 is showing total number of susceptible individuals S(t) with initial variable condition 3000000. Control parameters used are as in Tables (2) and (3). Figure 2, when the control parameters are $\alpha = \gamma = \sigma$ =0.025 while Figure 3, $\alpha = \gamma = \sigma$ = 0.00, was assumed.

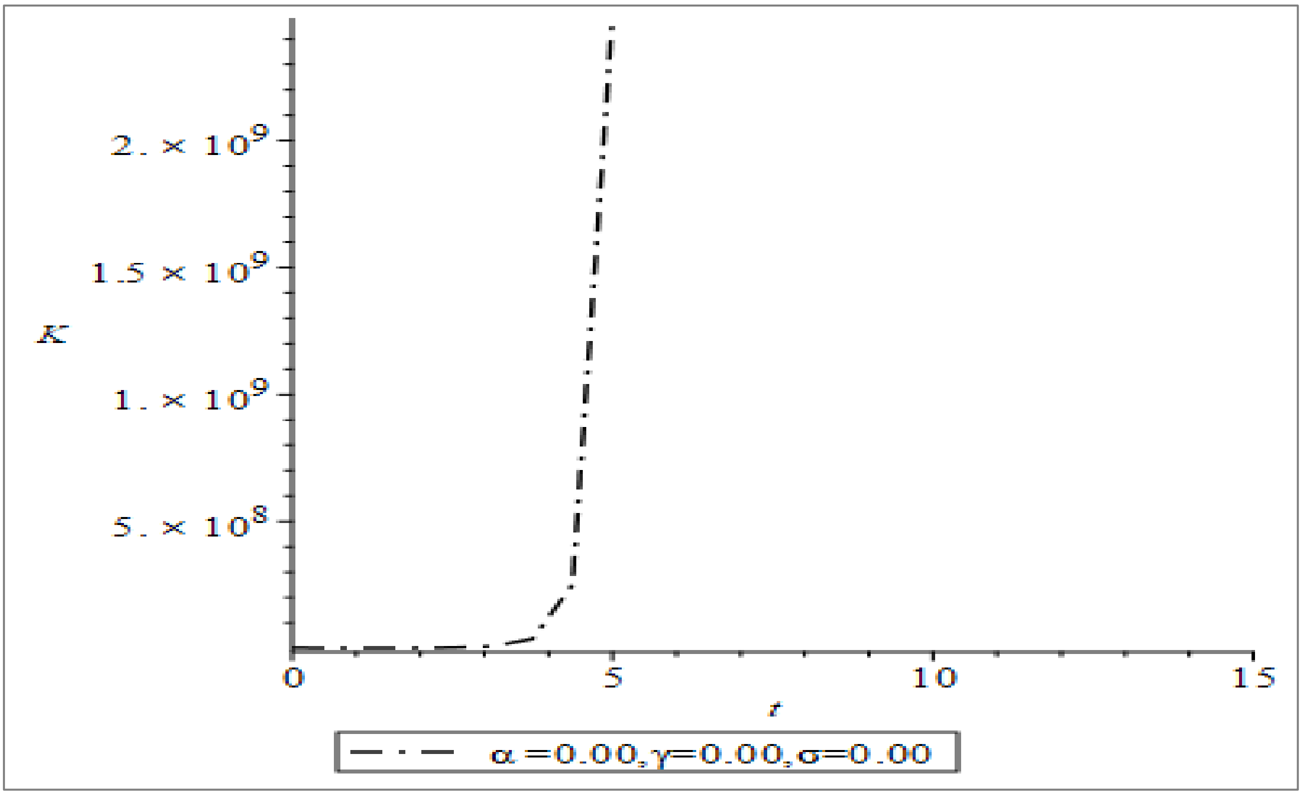

Figure 4: Graph of kidnappers without control:

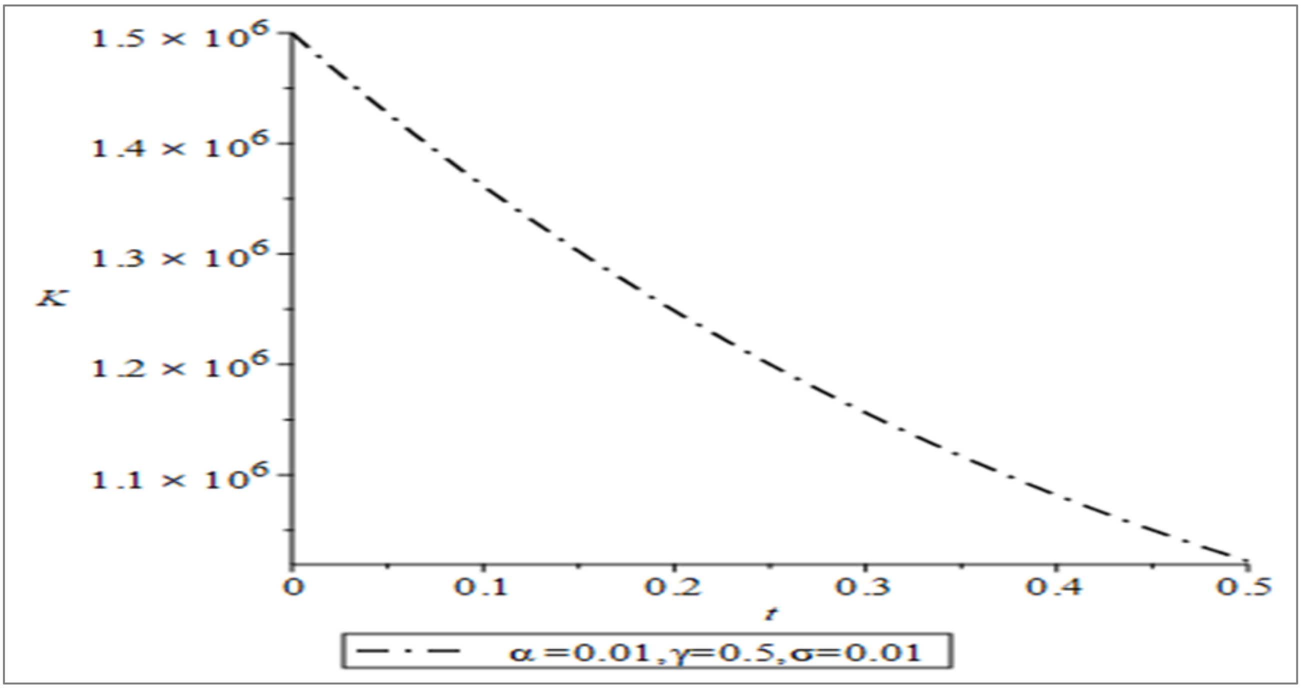

Figure 5: Graph of kidnappers with control

Figure 4 is showing total number of kidnappers K(t) with initial condition15000. Control parameters used are in Table (2) and (3), Figure 4, $\alpha = \gamma = \sigma$ = 0.00, i.e. all the control strategies are zero, while Figure 5, $\alpha = \gamma = \sigma$ = 0.5, that is when there is medium control.

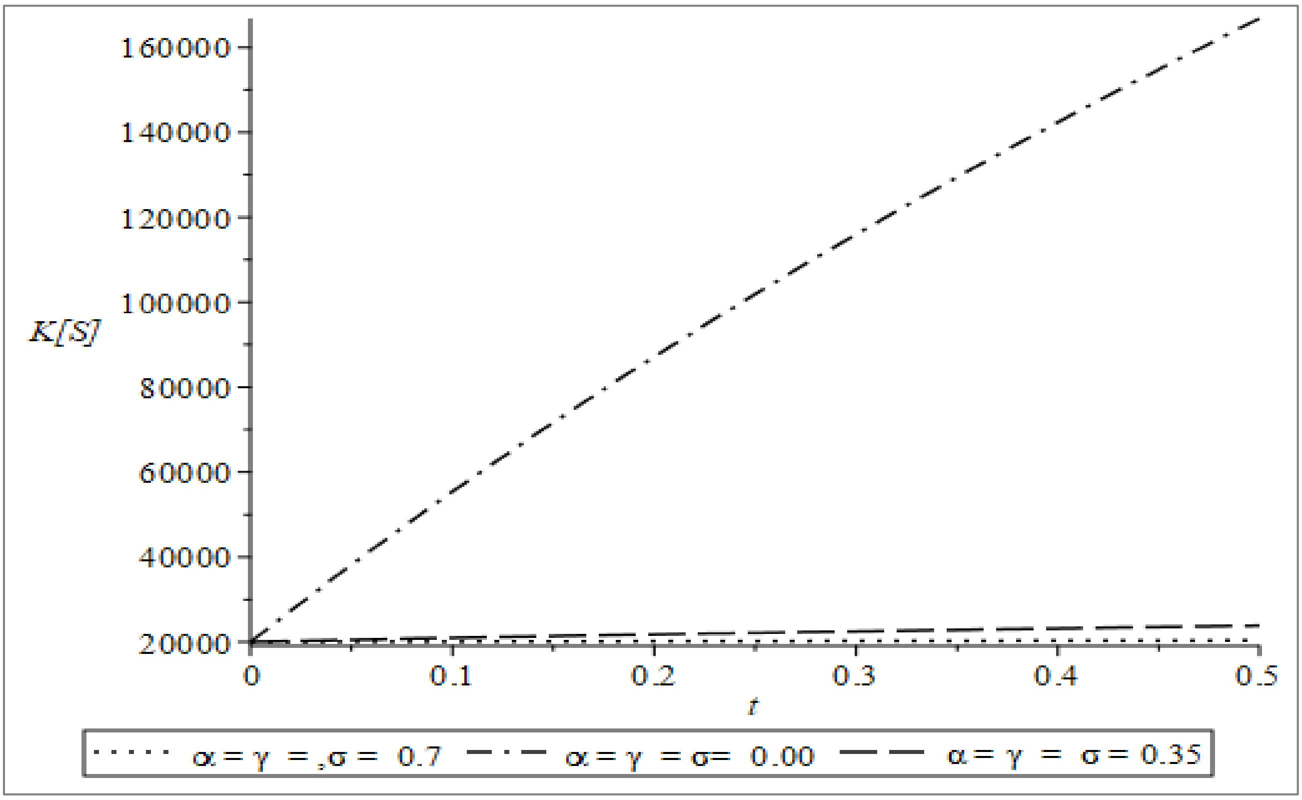

Figure 6: The graph of kidnappers Sponsors, Parameters values used as in Table 2 with $\alpha = \gamma = \sigma$ = 0.7, $\alpha = \gamma = \sigma$ = 0.00 and $\alpha = \gamma = \sigma$ = 0.035.

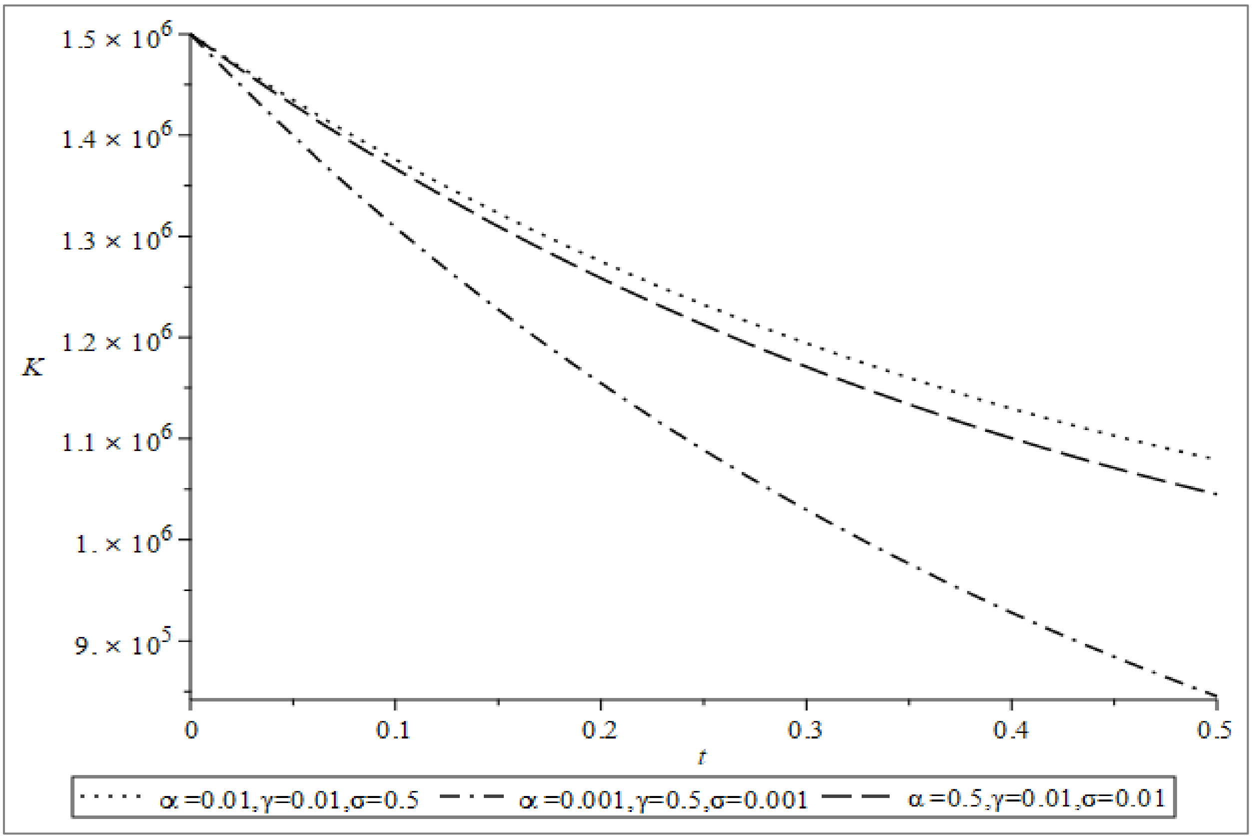

Figure 7: Effect of killing as well as imprisoning of kidnappers, and preventing of susceptible individuals S(t), from joining kidnappers.

Figure 7 is the graph of kidnappers by comparing the effects of controls where we compared ($\alpha = 0.001$, $\gamma = 0.01$, $\sigma = 0.5$) , ($\alpha = 0.01$, $\gamma = 0.5$, $\sigma = 0.01$) and finally ($\alpha = 0.5$, $\gamma = 0.01$, $\sigma = 0.1$) .

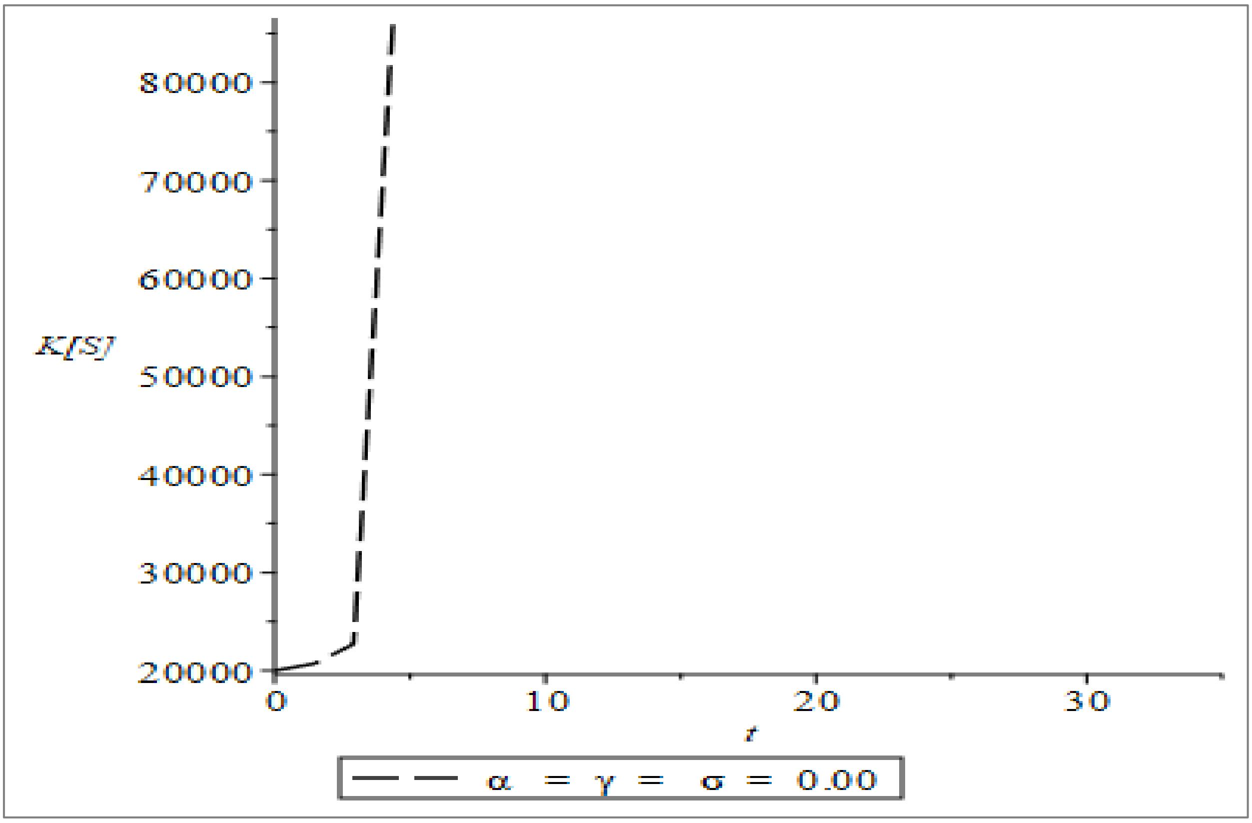

Figure 8: Total number of kidnappers sponsors with initial variables conditions: 20000. control parameters used are as in Tables 2 and 3, with $\alpha = \gamma = \sigma$ 0.00.

DISCUSSION OF NUMERICAL RESULTS:

Figures 2 and 3 illustrate the impact of control measures on susceptible individuals. It is observed that, with the high control measures, the population of susceptible individuals will increase over time t, similarly, without any control measure, the population of susceptible individuals will converge to almost kidnapping-free. Also, we observed in Figures 4 and 5 that as soon as there is no control, the solution profiles converges to endemic, while in Figure 6, in which the control measures were highly taken, the population of kidnappers decreased drastically.

Figure 6 illustrates the impact of control measures on kidnappers' sponsors. It is clear that any of the three (3) control strategies (rehabilitation, killing, preventing susceptible from joining banditry) have a positive effect on controlling the kidnappers' sponsors, but all can lead to a stable free state. Whenever the controls are zero, the population of kidnapper sponsors continues to persist, but when there is high control, the population of kidnapper sponsors will converge to kidnapping-free.

In Figure 7, we observed that taking kidnappers' sponsors to jail is the best strategy to curve sponsoring kidnapping activities, preventing the susceptible individuals from joining them and then preventing susceptible individuals from becoming informants. Lastly, in Figure 6, it is clear that the number of kidnappers' sponsors will increase as soon as any control measures are absent.

CONCLUSION.

This study demonstrates that targeted control measures, such as the elimination and incarceration of kidnappers, can effectively reduce kidnapping incidents.

Kidnappers have suffered a serious setback as a result of government actions. A few factors are among the notable factors contributing to kidnapping in Nigeria. Politicians, farmer-header conflicts, and gold miners. The results of this study, however, might be extended to other domains of study to investigate the prevalence of kidnappers who seriously endanger public safety in different regions of Nigeria. Thus, plans are being created to deal with this.

RECOMMENDATIONS.

1. It is impossible to anguish the kidnappers group without reducing the strength of their organization by either killing or taking them to jail.
2. Security agents should increase their operations to search and deal with informants.
3. NGOs should collaborate with the Government to implement policies for youth to reduce poverty and unemployment in the country.
4. The government should increase security agencies to tackle the kidnappers.
5. The government should not consider kidnappers only but also prevent susceptible individuals from joining kidnappers or informants.
6. The government should enhance security measures, focus on disrupting the kidnapping network, and rehabilitate affected individuals.

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