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ORIGINAL RESEARCH ARTICLE

On Power Graph Representation of Γ_1-nonderanged Permutation Group

Anas Yusuf[1] and Almustapha Umar[1]*
[1]Department Of Mathematics, Faculty of Science, Federal University, Birnin Kebbi, Kebbi State, Nigeria

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ABSTRACT
A Γ_1-nonderanged permutation group $G_P^{\Gamma_1}$ ($p \geq 5$ and p is prime) is a permutation group such that $G_P^{\Gamma_1} = \{\omega_i : 1 \leq i \leq p-1\}$ where $\omega_i = ((1+0i)(1+1i)_{mP}(1+2i)_{mP} \dots (1+(P-I)i)_{mP})$. In this paper, an undirected power graph representation of Γ_1-nonderanged permutation group $G_P^{\Gamma_1}$ denoted by $\Gamma_P(G_P^{\Gamma_1})$ has been studied. It was proved, among other things, that the graph $\Gamma_P(G_P^{\Gamma_1})$ is connected for any $p \geq 5$ and is neither regular nor complete except at $p = 5$. Also, the maximum degree of $\Gamma_P(G_P^{\Gamma_1})$ is $p-2$, and the girth is three (3) for any $p \geq 5$, while the diameter is one (1) for $p = 5$ and two (2) for $p > 5$. Furthermore, the central vertex of the graph was proved to be ω_1 for any $p \geq 5$, and while the peripheral vertex is ω_{p-1} for p > 5. Lastly, the adjacency matrix of some selected graphs and their graphical representations were given to support our findings.

INTRODUCTION

The concept of permutation may be traced to the work of Lagrange in 1770 to find the algebraic solutions of polynomial equations. This concept can be described as how numbers can be arranged or the rearrangement of elements of an ordered set mapped to itself in a one-to-one correspondence. A permutation group *G*, according to (Beaumont and Peterson, 1955), is a set of arrangements of *G* that form a group under function composition, and this type of group forms one of the oldest parts of group theory (Dixon and Mortimer, 1996). In the study of group theory, the beauty and applications are in the algebraic properties; thus, this explains the vast interest of algebraic theorists in the properties of mathematical structures such as generalized groups, symmetry groups, permutation groups, etc. Similarly, using some algebraic properties, (Adeniran *et al.,* 2011) explore the properties of the generalized groups to show that some results that are true in classical groups are either generally true or only true in some special types of generalized groups; for instance, it was shown that a Bol groupoid and a Bol quasigroup could be constructed using a non-Abelian generalized group. Similarly, (Grigorchuk and Medynets, 2011) studied a topological full group and noted that the structure is similar to a union of permutational wreath products of finite groups. Over the years, studies on deranged permutation groups have been carried out; for instance, (Calkin *et al.,* 2000) looked at the Lampert-Slater sequence using a free mapping of $[n+1]$, which omits exactly k elements from its image and discovered that the sequence exhibits oscillatory behaviour but little could be seen of non-deranged permutation, however, Γ_1- nonderanged permutation group $G_P^{\Gamma_1}$ was introduced by Ibrahim (Ejima and Aremu, 2016); they constructed and studied the representation of the group and further noted that the group is an FG-Module. Similarly, (Garba *et al.,* 2017) showed that the Young tableaux of the Γ_1 nonderanged permutation group $G_P^{\Gamma_1}$ is nonstandard. Consolidating on the work of (Ibrahim *et al.,* 2016), (Suleiman *et al.,* 2020) used composition operation to construct some non-deranged permutation groups, providing a new method of constructing permutation groups from existing ones. Also, using the embracing sum, the ascent block of Γ_1- nonderanged permutation group $G_P^{\Gamma_1}$ was studied by Ibrahim (Ibrahim and Ibrahim, 2022). Garba et al. (2018) studied some topological properties of Γ_1-nonderanged permutation group by constructing a topology on the structure and proved that the topology is bihomogeneous. More recently, (Yusuf and Ejima, 2023) gives an extension of the structure and show that the extended version of Γ_1-nonderanged permutation group is a commutative ring with identity and a vector space.

The main mathematical tools for studying an object's symmetries are groups and are usually related to graph

Correspondence: Almustapha Umar. Department Of Mathematics, Faculty of Science, Federal University, Birnin Kebbi, P.M.B. 1157, Kebbi State, Nigeria. ✉ almustapha.umar@fubk.edu.ng.
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automorphisms. In the literature, several researchers investigate the algebraic properties of groups using the associated graph, making it arguably the most famous, exciting, and productive area of algebraic graph theory. One of the representations of groups by graphs is the power graphs, which were first used in 2002 by Kelarev and Quinn (Kelarev and Quinn 2002) as the directed power graph of a semi group S with vertex set S in which there is an arc from two points x to y if and only $if\ x \leq\ y\ and\ y\ =\ x^n$ for positive integer n. Motivated by this, (Chakrabarty *et al.,* 2009) defined the undirected power graph $\Gamma_P(G)$ of a group G with vertex set $V\ (\Gamma_P(G)\ )\ =\ G$ and two distinct vertices x, $y\ \in G$ are adjacent in $\Gamma_P(G)$ if and only if either $x^n\ =\ y\ or\ y^m\ =\ x$, where n and m are integers greater than or equal to 2, using the same definition given by (Chakrabarty *et al.,* 2009) and (Chelvam and Sattanathan 2013) characterize certain classes of power graphs of finite Abelian groups.

In this paper, we implore the definition of a power graph given by (Chakrabarty *et al.,* 2009). The order of Γ_P is the number of vertices in the graph, and its maximum and minimum degrees will be denoted by $\Delta(\Gamma_P)$ and $\delta(\Gamma_P)$ respectively, if the degree of all the vertices in graph Γ_P are the same then the graph is regular. The length of the smallest cycle in a graph Γ is referred to as its girth, which is denoted by $gr(\Gamma_P)$. The distance between two vertices u and v in a graph is the length of the shortest path from u to v denoted by $d(u,v)$. The eccentricity of a vertex v is the maximum distance from it to any other vertex and is denoted by $e(v)$, the radius of the graph Γ_P denoted by $rad(\Gamma_P)$ is the minimum eccentricity, and the diameter denoted by $diam(\Gamma_P)$ is the maximum eccentricity. A vertex of maximum eccentricity is called the peripheral vertex if the vertex whose eccentricity is equal to the diameter. If the eccentricity of the vertex equals to the radius of the graph, then it is called the central vertex.

Definition 1 (Γ_1-nonderanged permutation group $G_P^{\Gamma_1}$): Let Γ_1 be a non-empty, totally ordered, and finite subset of N. Let p be a prime number greater than or equal to 5 such that

$$G_P^{\Gamma_1} = \{\omega_1, \dots \omega_{P-1}\}$$

where ω_i is a bijection on Γ_1- written in the form

$\omega_i\ =\ (1+0\mathrm{i})(1\ +\ 1i)mP\ (1\ +\ 2i)mP \dots (1\ +\ (p\ -\ 1)i)mP$ with $mP\ =\ modulo\ p$.

Then, G^{Γ_1} is said to be a Γ_1-nonderanged permutations.

Remark 1 G^{Γ_1} together with a natural permutation composition is a group. Thus, G^{Γ_1} is called a Γ_1-nonderanged permutation group.

Definition 2: The n^{th} successor in a cycle ω_i is given by

$$\omega_i(n)\ =\ (1\ +\ (n\ -\ 1)i)mP$$

where $1\ \leq\ n\ \leq\ p\ and\ 1\ \leq\ i\ \leq\ p\ -\ 1$.

Example 1 $For\ p\ =\ 5, G^{\Gamma_1}\ =\ \{(12345),(13524),(14253),(15432)\}$,

where $\omega_1\ =\ (12345), \omega_2\ =\ (13524), \omega_3\ =\ (14253), \omega_4\ =\ (15432)$, each with length 5.

RESULTS

Proposition 1 Let $G_P^{\Gamma_1}$ be a Γ_1-nonderanged permutation group and $\omega_i\ \omega_j \in G_P^{\Gamma_1}$. The multiplication '·' (composition permutation) in G^{Γ_1} is an endomorphism given by

$$\omega_i\ .\ \omega_j = \omega_{(i \times j)mP}$$

Proof: Let $\omega_i\ \omega_j \in G_P^{\Gamma_1}$, by Definition (3), we have

$$\omega_i = ((1+0\mathrm{i})(1+1i)_{mP}(1+2i)_{mP} \dots\ (1+(p-1)i)_{mP}) \quad (1)$$

$$\omega_j = ((1+0\mathrm{j})(1+1j)_{mP}(1+2j)_{mP} \dots\ (1+(p-1)j)_{mP}) \quad (2)$$

But $\omega_i\ .\ \omega_j = \omega_i(\omega_j)$ Thus, we have

$$\omega_i\ .\ \omega_j = \omega_i((1+0\mathrm{i})(1+1i)_{mP}(1+2i)_{mP} \dots\ (1+(p-1)i)_{mP}$$

$$= (\omega_i(1+0\mathrm{j})(1+1j)_{mP}, \omega_i(1+2j)_{mP,} \dots\ \omega_i(1+(p-1)j)_{mP}) \quad (3)$$

Using the n^{th} successor in $G_P^{\Gamma_1}$ which is given by $\omega_i(n) = (1 + (n - 1)i)mP$ we have

$$\omega_i(1) = 1$$

$$\omega_i(1 + j) = (1 + (1 + j - 1)i)_{mP} = \left(1 + (i \times j)\right)_{mP}$$

$$\omega_i(1 + 2j) = (1 + (1 + 2j - 1)i)_{mP} = (1 + 2(i \times j))_{mP}$$

$$\omega_i(1 + (p - 1)j) = (1 + (1 + (p - 1)j - 1)i)_{mP} = (1 + (p - 1)(i \times j))_{mP}$$

Substituting the above equations into Equation (3) we have

$$\omega_i . \omega_j = ((1), (1 + (i \times j))_{mP}, \left(1 + 2(i \times j)\right)_{mP}, \dots, \left(1 + (p - 1)(i \times j)\right)_{mP}) = \omega(i \times j)_{mP}$$

Example 2: For P = 5, $G_P^{\Gamma_1} = \{(12345), (13524), (14253), (15432)\}$,

where $\omega_1 = (12345)$, $\omega_2 = (13524)$, $\omega_3 = (14253)$, $\omega_4 = (15432)$,

To multiply two cycles, say ω_2, ω_3 in $G_P^{\Gamma_1}$. We have by Proposition (1) that $\omega_2 . \omega_3 = \omega_{(2\times3)_{m5}} = \omega_{(6)m5} = \omega_1$. This can be viewed by using two line notation as below

$$\omega_2 . \omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = \omega_1$$

Also, for p = 7,

$$G_P^{\Gamma_1} = \{(1234567), (1357246), (1473625), (1526374), (1642753), (1765432)\},$$

let us take ω_4 and ω_6 which by Proposition (1) is $\omega_4 . \omega_6 = \omega_{(4\times6)_{m7}} = \omega_{(24)_{m7}} = \omega_3$.

This also can be viewed in two line notation a

$$\omega_4 . \omega_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 2 & 6 & 3 & 7 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 7 & 3 & 6 & 2 & 5 \end{pmatrix} = \omega_3$$

Lemma 1: Let $G_P^{\Gamma_1}$ be a Γ_1-nonderanged permutation group and $\omega_i \in G_P^{\Gamma_1}$. Then, $\omega_i^{p-1} = \omega_1$ *and* $\omega_{p-1}^2 = \omega_1$.

Proof: Let $\omega_i \in G_P^{\Gamma_1}$. Then,

$$\omega_i^{P-1} = \omega_i^P . \omega_i^{-1}$$

$$= \underbrace{(\omega_i \times \omega_i \times \dots \times \omega_i)}_{p\ times} . \omega_i^{-1}$$

$$= \omega_{(i\times i\times\dots\times i)mP} . \omega_i^{-1}$$

$$= \omega_i . \omega_i^{-1}$$

$$= \omega_{(i\times i^{-1})mP}$$

$$= \omega_{i^0}$$

$$= \omega_1$$

Also, to show that $\omega_{P-1}^2 = \omega_1$, observes that

$$\omega_{p-1}^2 = \omega_{P-1}.\omega_{P-1}$$

$$= \omega_{(P-1)\times(P-1)mP}$$

$$= \omega_{(P^2-2P+1)mP}$$

$$= \omega_1$$

Lemma 2: Let $\omega_i \in G_P^{\Gamma_1}$. For $i = 4$, the smallest positive integer n for which $\omega_i^n = \omega_1$ is $\frac{P-1}{2}$.

Proof: Let $i = 4$, it suffices to show that $\omega_4^{\frac{P-1}{2}} = \omega_1$

$$\omega_4^{\frac{P-1}{2}} = \omega_4^{\frac{P}{2}} . \omega_4^{\frac{-1}{2}}$$

$$= \omega_{4^{\frac{P}{2}}} . \omega_{4^{\frac{-1}{2}}}$$

$$= \omega_{2^p} . \omega_{2^{-1}}$$

$$= \underbrace{(\omega_2 \times \omega_2 \times \ldots \times \omega_2)}_{p\ times} . \omega_2^{-1}$$

$$= \omega_{(2\times 2\times \ldots \times 2) mod P} . \omega_2^{-1}$$

$$= \omega_2 . \omega_2^{-1}$$

$$= \omega_{(2\times 2^{-1}) mod P}$$

$$= \omega_{2^0}$$

$$= \omega_1$$

Remark 2: From Lemma (2) ahead, it can be seen that the number of residues of ω_4 are $\left(\frac{P-1}{2}\right) - 1$ excluding ω_4 itself.

Example 3: For $p = 5$, to check the number of residues of ω_4, we use the fact that $\left(\frac{P-1}{2}\right) - 1 = \left(\frac{5-1}{2}\right) - 1 = 1$, which means there is only one residue of ω_4 for $p = 5$ excluding ω_4. Hence, $\omega_4^{\frac{P-1}{2}} = \omega_4^2 = \omega_{16}$ mod $5 = \omega_1$ is the only residue of ω_4. For $p = 7$, the number of residues of ω_4 is $\left(\frac{P-1}{2}\right) - 1 = \left(\frac{7-1}{2}\right) - 1 = 2$. Which means there are two residues of ω_4. This can be seen as $\omega_4^{\frac{7-1}{2}} = \omega_4^3 = \omega_{64}\ mod\ 7 = \omega_1$ and $\omega_4^2 = \omega_{16}\ mod\ 7 = \omega_2$.

Proposition 2: Let $G_P^{\Gamma_1}$ be a Γ_1-nonderanged permutation group and $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. Then, for any vertex $\omega_i (2 \leq i \leq p-1) \in \Gamma_P(G_P^{\Gamma_1})$ there is always an edge linking it to the vertex $\omega_i \in \Gamma_P(G_P^{\Gamma_1})$.

Proof: Let ω_i be any vertex in $\Gamma_P(G_P^{\Gamma_1})$. We need to show by definition that there exists an $n \geq 2$ such that $\omega_i^n = \omega_i$ for $i \in [2, p-1]$. To do this, let $n = p - 1$, then by Lemma (1) $\omega_i^{p-1} = \omega_i$ for any $i \in [2, p-1]$. This means that there is always an edge between the vertex ω_i for $i \in [2, p-1]$ and vertex ω1 in a power graph of $G_P^{\Gamma_1}$.

Proposition 3: The maximum degree in a power graph Γ_P of Γ_1-nonderanged permutation group is $p - 2$ $i.e$ $\Delta(\Gamma_P(G_P^{\Gamma_1})) = p - 2$.

Proof: From Proposition (2), we know that there is always an edge between the vertex ω_i for $i \in [2, p-1]$ and vertex ω_1. Since there are $p - 1$ vertices in $\Gamma_P(G_P^{\Gamma_1})$ we have that $\Delta(\Gamma_P(G_P^{\Gamma_1})) = p - 2$.

Proposition 4: Let $G_P^{\Gamma_1}$ be a Γ_1-nonderanged permutation group and $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. For $p \neq 5$, there exists an $\omega_i \in \Gamma_P(G_P^{\Gamma_1})$ such that $deg(\omega_i) < p - 2$.

Proof: Let $p > 5$, to show that $deg(\omega_i) < p - 2$ for some $i \in [2, p-1]$, it suffices to show that there is at least 2 elements in $G_P^{\Gamma_1}$ such that $\omega_i^n = \omega_1$ for $n < p - 1$. By Lemma (1), we have that $\omega_{P-1}^2 = \omega_1$ and by Lemma (2), we have that $\omega_4^{\frac{P-1}{2}} = \omega_1$. Hence $deg(\omega_i) < p - 2$ for some $i \in [2, p-1]$.

Theorem 1: Let $\Gamma_P(G_P^{\Gamma_1})$be a power graph of $G_P^{\Gamma_1}$. Then, for each prime $p \geq 5$, $\Gamma_P(G_P^{\Gamma_1})$ is a connected graph.

Proof: To show that $\Gamma_P(G_P^{\Gamma_1})$ is connected. It suffices to show that there is a path between any two distinct vertices in $\Gamma_P(G_P^{\Gamma_1})$. Let $\omega_i, \omega_1 \in G_P^{\Gamma_1}$ be such $that\ 2 \leq i \leq p - 1$. Then by Lemma (1) $\omega_i^{p-1} = \omega_1$ which implies that there is an edge linking ω_1 and any other vertex ω_i for $i \in [2, p-1]$. Thus, the graph $\Gamma_P(G_P^{\Gamma_1})$ is connected.

Proposition 5: Let $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. Then, for each p ≥ 5, p prime, the girth of $\Gamma_P(G_P^{\Gamma_1}) = 3$.

Proof: We need to show that the length of the shortest cycle of $\Gamma_P\left(G_P^{\Gamma_1}\right) = 3$. To do this, we must demonstrate that there is no cycle of length 2 and that a cycle of length 3 exists.

Case 1: No cycle of length 2 since we are considering an undirected power graph $\Gamma_P(G_P^{\Gamma_1})$ then, there is no cycle of length 2, since $(\omega_i, \omega_j) \in E(G_P^{\Gamma_1})$ even if $\omega_i^n = \omega_j$ and $\omega_i^n = \omega_i$.

Case II: Existence of cycle of length 3. Consider an arbitrary element ω_i in $\Gamma_P(G_P^{\Gamma_1})$). We want to find a cycle of length 3 starting from ω_i . Assume that there exist an edge between ω_i and ω_j for $i \neq j$ then we can find a path ω_i •−• ω_1 •−• ω_j since $\omega_i^{p-1} = \omega_1$ by Lemma (1). But since ω_i is connected to ωj we have that ω_i •−• ω_1 •−• ω_j •−• ω_i which is a cycle of length 3.

Theorem 2: Let $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. Then, $\Gamma_P\left(G_P^{\Gamma_1}\right)$ is a complete graph for $p = 5$ with each vertex is of degree 3.

Proof: For $p = 5$, $G_P^{\Gamma_1} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. To get the edges, observe that:

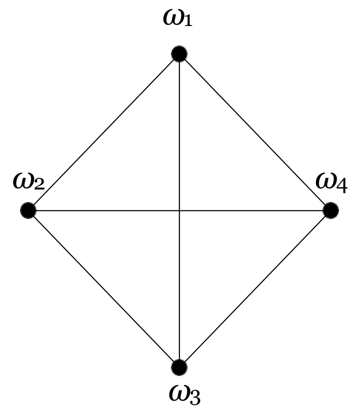
$\omega_2^2 = \omega_4, \omega_2^3 = \omega_3, \omega_2^4 = \omega_1$

$\omega_3^2 = \omega_4, \omega_3^3 = \omega_2, \omega_3^4 = \omega_1$

$\omega_4^2 = \omega_1$

Thus, the set of edges will be

$E\left(\Gamma_P\left(G_P^{\Gamma_1}\right)\right) = \{(\omega_2, \omega_4), (\omega_2, \omega_3), (\omega_2, \omega_1), (\omega_3, \omega_4), (\omega_3, \omega_1), (\omega_4, \omega_1)\}$. This can be pictured graphically as



Since every vertex ω_i is connected, we conclude that $\Gamma_P(G_5^{\Gamma_1})$ is a complete graph with degree of each vertex as 3.

Theorem 3: Let $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. Then, $\Gamma_P(G_P^{\Gamma_1})$ is not regular except for $p = 5$, where p is prime.

Proof: For $p = 5$, the degree of each vertex in the power graph $\Gamma_P(G_P^{\Gamma_1})$ is always 3 as can be seen by Theorem (2), thus, the graph is regular at $p = 5$.

Assume that p is prime and $p > 5$, then by Proposition (4), we have that the maximum degree in $\Gamma_P(G_P^{\Gamma_1}) = p - 2$ and by Proposition (3), we saw that there exists a vertex ω_i in $\Gamma_P(G_P^{\Gamma_1})$ such that the $deg(\omega_i) < p - 2$. Thus, $\Gamma_P(G_P^{\Gamma_1})$ is not a regular since not all the vertices have the same degree size.

Theorem 4: Let $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. For each $p \geq 5$, the diameter of $\Gamma_P(G_P^{\Gamma_1})$ is given by

$$diam\left(\Gamma_P\left(G_P^{\Gamma_1}\right)\right) = \begin{cases} 1, for\ p = 5 \\ 2, for\ p > 5 \end{cases}$$

Proof: From the definition of a diameter of a graph, it suffices to find the longest shortest path between any two vertices. For $p = 5$ we know by Theorem (2) that the power graph of $G_P^{\Gamma_1}$ is complete; hence, the diameter is 1. Assume that $p > 5$ then, the power graph is not regular by Theorem (3), hence not complete, which means there a vertices $\omega_i, \omega_j, \omega_k \in \Gamma_P\left(G_P^{\Gamma_1}\right)$ such that $\omega_i\, \omega_j, \omega_j \omega_k \in E(\Gamma_P\left(G_P^{\Gamma_1}\right))$ but $\omega_i \omega_k \notin E(\Gamma_P\left(G_P^{\Gamma_1}\right))$.

Consider two arbitrary elements $\omega_i , \omega_j \in G_P^{\Gamma_1}$, the edge between $\omega_i \; and \; \omega_j$ exist if and only if $\omega_i^n = \omega_j$ and $\omega_i^n = \omega_i$where $i \neq j$ and n is any positive integer greater than or equals to 2. Take any 10 element $\omega_i , \omega_j \in G_P^{\Gamma_1}$ then, for $n = p - 1, \omega_i^{p-1} = \omega_1$ by Lemma (1) (That is, there is always an edge between any $\omega_i \; to \; \omega_1$). Assume that there is no edge between $\omega_i \; and \; \omega_k$ $(2 \leq i, k \leq p - 1)$ with $\omega_i \neq \omega_k$, then we can find a path of length 2 traversing through $\omega_1, i.e \;\; \omega_i \; \bullet - \bullet \; \omega_1 \; \bullet - \bullet \; \omega_k$ for any choice of ω_i and ω_k. This implies that the diameter of $\Gamma_P(G_P^{\Gamma_1}) = 2 \; for \; p > 5$.

Theorem 5: Let $\Gamma_P(G_P^{\Gamma_1})$ be a power graph of $G_P^{\Gamma_1}$. Then, the sum of degree of all vertices in $\Gamma_P(G_P^{\Gamma_1})$ is $(p - 1)(p - 2) \; for \; p = 5$ and less than $(p - 1)(p - 2) \; for \; p > 5$.

Proof: For $p = 5$, the power graph of $G_P^{\Gamma_1}$ is complete, and $deg(\omega i) = p - 2$ by Proposition (3). Since there are $p - 1$ element in $G_P^{\Gamma_1}$ then

$$\sum_{i=1}^{P-1} \deg(\omega_i) = (p - 2) + (p - 2) + \ldots + (p - 2) upto \; (p-1) times$$

$$= (p - 1)(p - 2)$$

Let $p > 5$, observes that if $\Gamma_P\left(G_P^{\Gamma_1}\right)$ is complete then $\sum_{i=1}^{P-1} deg(\omega_i) \; (p - 1)(p - 2)$ holds, but for $p \neq 5$ we know by Proposition (4) that power graph is not complete and that $deg(\omega_{p-1}) < (p - 2)$ Thus, $\sum_{i=1}^{P-1} deg(\omega_i) < (p - 1)(p - 2)$.

Proposition 6: Let $\Gamma_P\left(G_P^{\Gamma_1}\right)$ be a power graph of $G_P^{\Gamma_1}$. The vertex $\omega_1 \in \Gamma_P\left(G_P^{\Gamma_1}\right)$ is said to be the central vertex for any $p \geq 5$.

Proof: It suffices to show that $e(\omega_1) = rad(\Gamma_P\left(G_P^{\Gamma_1}\right))$. Observe that $\omega_i^{p-1} = \omega_1$ for any $\omega_i \in \Gamma_P\left(G_P^{\Gamma_1}\right)$ where $i \in [2, p - 1]$ by Lemma (1), that means there is always an edge linking $\omega_i \; to \; \omega_1$. Thus, $d(\omega_1 , \omega_i) = 1$ and hence $e(\omega_1) = max\{d(\omega_1 , \omega_i) | \; \omega_i \in \Gamma_P\left(G_P^{\Gamma_1}\right)\} = 1$ and also $rad(\Gamma_P\left(G_P^{\Gamma_1}\right)) = 1$ since the minimum eccentricity of the graph is also 1. Thus, $e(\omega_1) = rad(\Gamma_P\left(G_P^{\Gamma_1}\right)). = 1$.

Proposition 7: Let $\Gamma_P\left(G_P^{\Gamma_1}\right)$ be a power graph of $G_P^{\Gamma_1}$. The vertex $\omega_{p-1} \in \Gamma_P\left(G_P^{\Gamma_1}\right)$ is said to be the peripheral vertex for any p > 5.

Proof: It suffices to show that $e(\omega_{P-1}) = diam(\Gamma_P\left(G_P^{\Gamma_1}\right)$. Observes that $\omega_i^{p-1} = \omega_1$ for any $\omega_i \in \Gamma_P\left(G_P^{\Gamma_1}\right)$ where $i \in [2, p - 1]$ by Lemma (1), that means there is always an edge linking ω_i to ω_1. Thus, $d(\omega_1, \omega_i) = 1$ and hence $e(\omega_1) = max\{d(\omega_1 , \omega_i) | \; \omega_i \in \Gamma_P\left(G_P^{\Gamma_1}\right)\} = 1$ and also $rad(\Gamma_P\left(G_P^{\Gamma_1}\right))) = 1$ since the minimum eccentricity of the graph is also 1. Thus, $e(\omega_1) = rad(\Gamma_P\left(G_P^{\Gamma_1}\right)) = 1$.

ADJACENCY MATRIX OF A POWER GRAPH OF $G_5^{\Gamma_1}$, $G_7^{\Gamma_1}$, $G_{11}^{\Gamma_1}$ AND $G_{13}^{\Gamma_1}$

Below are some adjacency matrix of some selected graph together with their graphical representations.

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Adjacency matrix of $\Gamma_P\left(G_5^{\Gamma_1}\right)$

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix of $\Gamma_P\left(G_7^{\Gamma_1}\right)$

$$\begin{pmatrix} 0&1&1&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1&1&1\\ 1&1&0&1&1&1&1&1&1&0\\ 1&1&1&0&1&1&1&1&1&0\\ 1&1&1&1&0&1&1&1&1&0\\ 1&1&1&1&1&0&1&1&1&1\\ 1&1&1&1&1&1&0&1&1&1\\ 1&1&1&1&1&1&1&0&1&1\\ 1&1&1&1&1&1&1&1&0&0\\ 1&1&0&0&0&1&1&1&0&0 \end{pmatrix}$$

Adjacency matrix of $\boldsymbol{\Gamma_P\left(G_{11}^{\Gamma_1}\right)}$

$$\begin{pmatrix} 0&1&1&1&1&1&1&1&1&1&1&1\\ 1&0&1&1&1&1&1&1&1&1&1&1\\ 1&1&0&1&0&1&1&0&1&1&1&0\\ 1&1&1&0&0&1&1&0&1&1&1&1\\ 1&1&0&0&0&1&1&1&0&0&1&1\\ 1&1&1&1&1&0&1&1&1&1&1&1\\ 1&1&1&1&1&1&0&1&1&1&1&1\\ 1&1&0&0&1&1&1&0&0&0&1&1\\ 1&1&1&1&0&1&1&0&0&1&1&0\\ 1&1&1&1&0&1&1&0&1&0&1&1\\ 1&1&1&1&1&1&1&1&1&1&0&1\\ 1&1&0&1&1&1&1&1&0&1&1&0 \end{pmatrix}$$

Adjacency matrix of $\boldsymbol{\Gamma_P\left(G_{13}^{\Gamma_1}\right)}$

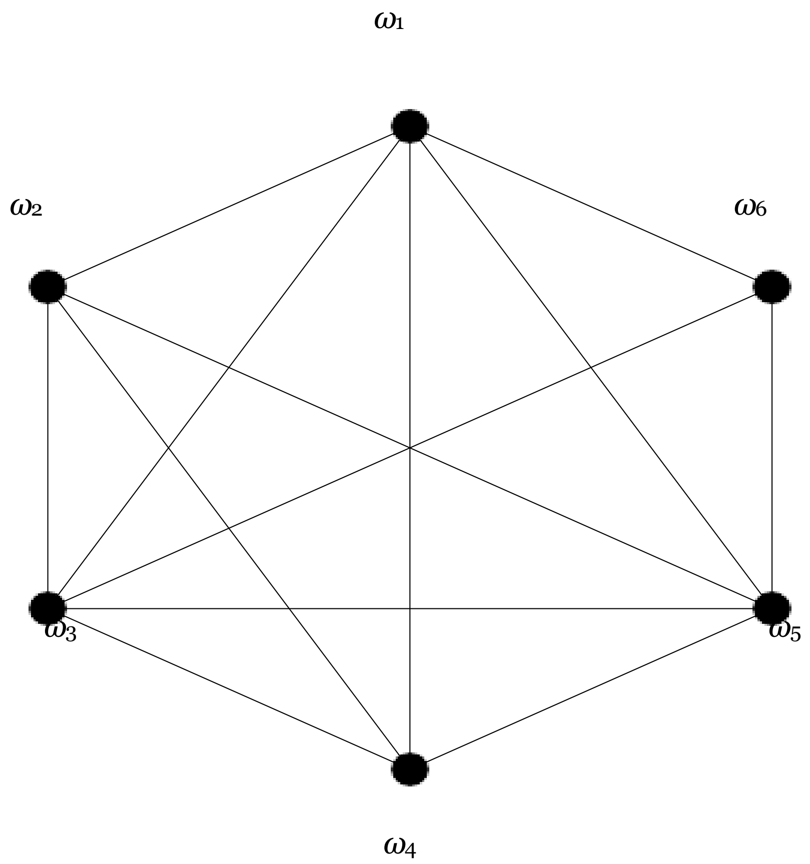


Figure 1. Power Graph representation of $\boldsymbol{G_7^{\Gamma_1}}$

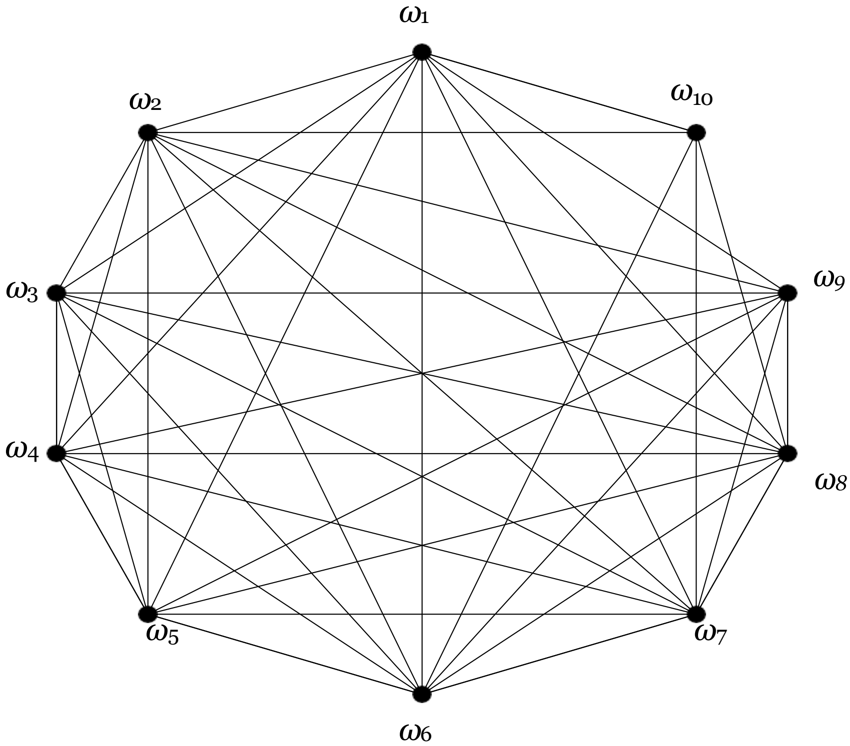


Figure 2. Power Graph representation of $G_{11}^{\Gamma_1}$

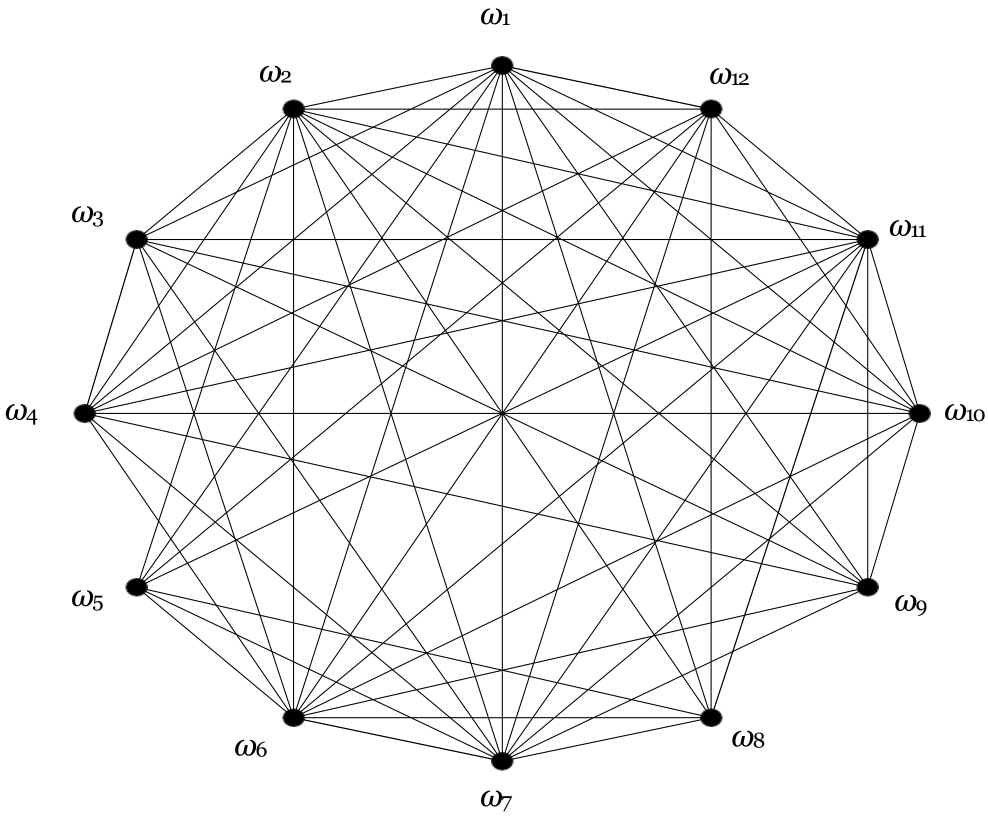


Figure 2. Power Graph representation of $G_{13}^{\Gamma_1}$

CONCLUSION

The power graph representation of Γ1-nonderanged permutation group $G_P^{\Gamma_1}$ was studied, and some of its properties, such as the connectivity and completeness, have been proved. It was observed that the power graph of Γ1-nonderanged permutation group $G_P^{\Gamma_1}$ is only regular for $p = 5$. The diameter and girth of the graph are all studied, while the diameter was found to be equal to 1 for $p = 5$ and 2 for any $p > 5$, the girth is 3 for any $p \geq 5$. The sum of all degrees in the power graph of $G_p^{\Gamma_1}$ is $(p-1)(p-2) for\ p = 5$, which is due to the regularity of graph at $p = 5$ and greater than $(p-1)(p-2) for\ p > 5$ since the graph is not regular at $p \neq 5$. In addition, the central vertex of the graph was found to be ω_1 and the peripheral vertex to be ω_{p-1}.

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